Decomposition of In-Situ Directional Wave Spectrum Applied to Improve Wave Model Simulations by the Data Assimilation Approach

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Abstract - **The purpose of this study is to enhance the accuracy of numerical wave forecasts through data assimilation. A sequential data assimilation scheme is modified to work with partitions of directional wave spectra. The performance of the system is investigated with respect to operational applications. Two typhoons, Kamei and Shanshan, occurring in 2006 around Taiwan sea areas, are used for a case study. It is verified that the proposed data assimilation increases the forecast accuracy; not only in terms to wave parmeters like wave height and periods. The results show, that after assimilation, the shape of directional spectra are much closer to independent observation.**

I. INTRODUCTION

Applying data assimilation to operational wave modeling is a quickly developing subject during the past 20 years. The reason, in part, lies in the near real-time availability of wave and wind observations, which numbers have drastically increased after the launch of earth-observing satellites. This situation has inspired many researchers to investigate the possibilities to include data assimilation methods in operational wave forecasting systems in order to improve the estimate of the current sea states.

Assimilation techniques for wave forecasting are commonly divided into sequential techniques (as in [1], [2]) and variation methods. Sequential methods are computationally cheap, and some success in improving the wave forecasts has been reported (as in [3]), which has led to the implementation of such a system into the operational wave analysis/forecast cycle at the European Centre for Medium-Range Weather Forecasts (ECMWF).

For the past decade, the most frequently used operational assimilation schemes have been single-time-level schemes, such as the optimal interpolation (OI) (as in [4], [5], [6] and [7]). This method is computationally fast, and therefore it is easily applicable to the online wave analysis/ forecasting conditions, but they suffer from some drawbacks. Unfortunately, forecast errors are often inhomogeneously distributed over the wave spectrum and limit the improvements obtained by wave height assimilation alone (as in [8]). Thus, some groups have challenged to use the SAR data (as in [5]). Although this may turn out to be useful for wave models of the world oceans in regional seas, the density of SAR observations is simply too low to have a serious impact on the wave analysis. Also, the spectral resolution of the SAR, which truncates waves shorter than about 100 m, is a larger problem for (partly) sheltered seas, where the average wavelengths are substantially shorter than those on the open ocean. However, for the regional seas, there is a good alternative to the SAR data. It is rather densely covered with pitch-and-roll buoys, which also measure spectral information. Moreover, their spectral characteristics are more suited to the typical wavelengths encountered and since they continuously measure at fixed position, they supply more data than satellites do for the region.

The aim of the present paper is to investigate the potential use of the spectral observations from pitch-and-roll buoys, which are reporting near real-time, for assimilation in an operational forecast system. The set-up of an optimal interpolation scheme, if only one buoy is available in the forecast domain, which is located in the deep ocean about 220 km away from the Taiwan coast is discussed, and the impact of assimilation on the wave analysis and forecast is quantified by comparing runs with and without assimilation for several typhoons in 2006.

II. DESCRIPTIONS OF THE SIMULATION REGION

The emphasis of this study is in the eastern Taiwan waters. In order to get detailed wave information in this region and to effectively simulate the wave field, a three level nesting scheme is applied, as shown in Fig. 1. The purpose of the larger region is to offer boundary values for the next finer layer. For this study we concentrate on the fine resolution layer 3 grid only. The SWAN wave model (as in [9]) was used for all layers.
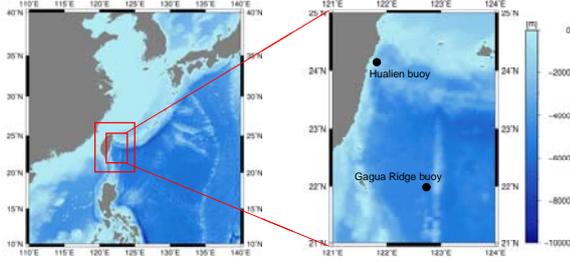
Fig. 1. Domain of the model and data buoy stations.

All SWAN model runs were forced by operational 1-hourly wind fields with a 0.5° resolution in longitude and latitude, provided by the Central Weather Bureau (CWB). The field were linearly interpolated in space and time.

Observed spectral data of Gagua Ridge buoy (122.78°E ; 22.01°N) were used for assimilation into the model. The Gagua Ridge buoy is located about 220 km east of Taiwan where the water depth is about 6000 m. For verifications the measurements at the Hualien buoy (Fig. 1) are used. The Hualien bouy site is moored near shore (about 1 km off-shore in a water depth of 21 m). Both pitch and role buoys are developed, manufactured and operated by the Coastal Ocean Monitoring Center (COMC) of National Cheng Kung University, assigned and supported by the CWB and report directional wave spectra every 1 hours. The Fast Fourier Transform (FFT) is used to obtain the full 2 dimensional wave spectrum (as in [10]).

III. AN INTRODUCTION OF THE DATA ASSIMILATION SCHEME

Optimal interpolation (OI, as in [11]) is a methodology to construct the analyzed significant wave height field. The optimal interpolation of partitions scheme (OI-P, as in [6] and [7]) was developed to assimilate spectral wave observations from pitch-and-roll buoys. Spectral partitioning (as in [12]) is a technique used to decompose a wave spectrum into the main wave systems, such as wind sea and/ or swell systems. Instead of specifying the main wave systems, a fairly accurate characterization of the spectrum can be given by specifying the wave energy of different frequencies and directions bands. In this study, the model simulated direction spectra is replaced by the observed datafrom data buoy, and the OI-P scheme is derived based on the procedure from OI formulas (Lionello et al., 1992) as follows.

The analyzed wave directional spectra at each point x_i, denoted as $S_A^i(f,\theta)$ listed below, is expressed as a linear combination of $S_P^i(f,\theta)$ indicating the first-guess results produced by the model and $S_O^k(f,\theta)$ (k=1, …, M_{obs}), the observation.

$$S_A^i(f,\theta) = S_P^i(f,\theta) + \sigma_P^i \sum_{k=1}^{M_{obs}} W_{ik} \frac{S_O^k(f,\theta) - S_P^k(f,\theta)}{\sigma_P^k} \quad (1)$$

Here σ_P^k is the root mean square error in the model prediction.

$$\sigma_P^k = \left\langle \left(S_P^k(f,\theta) - S_T^k(f,\theta) \right)^2 \right\rangle^{1/2} \quad (2)$$

where $S_T^k(f,\theta)$ represents the idealized true value of the wave directional spectra. The weights W_{ik} are chosen to minimize the root mean square error in the analysis of σ_A^k,

$$\sigma_A^k = \left\langle \left(S_A^k(f,\theta) - S_T^k(f,\theta) \right)^2 \right\rangle^{1/2}$$

The angle brackets indicate an average over a large number of realizations. Assuming that the errors in the model are unrelated with the errors in the measurements, the solution is

$$W_{ik} = \sum_{m=1}^{N_{obs}} P_{im} M_{mk}^{-1} \quad (3)$$

where the element of matrix M is the form as

$$M_{mk} = P_{mk} + O_{mk} \quad (4)$$

where P and O represent the error correlation matrices of prediction and observation, respectively (both are actually scaled with σ_P^i).

$$P_{mk} = \left\langle \frac{\left(S_P^m(f,\theta) - S_T^m(f,\theta) \right)\left(S_P^k(f,\theta) - S_T^k(f,\theta) \right)}{\sigma_P^m \sigma_P^k} \right\rangle \quad (5)$$

$$O_{mk} = \left\langle \frac{\left(S_O^m(f,\theta) - S_T^m(f,\theta) \right)\left(S_O^k(f,\theta) - S_T^k(f,\theta) \right)}{\sigma_P^m \sigma_P^k} \right\rangle \quad (6)$$

Therefore, the prediction error correlation matrix P and the observation error correlation matrix O must be clearly specified. This would, in practice, require the determination of statistics for both predictions and observations, which are presently unavailable. If the idealized true value is known, then, it is able to obtain the RMSE between the observations and first guess results. However, the error of any observation technique exists and thus we are unable to obtain the real observation. In this study, it is assumed that the prediction error correlation matrix is

$$P_{mk} = \exp\left(-\frac{|\bar{x}_m - \bar{x}_k|}{L_{\max}} \right) \quad (7)$$

With a radius of influence L_{max}=5 degree and that the observation errors $O_{mk} = \delta_{mk}\left(\sigma_O^m / \sigma_P^m \right) = \delta_{mk} R_m$ are random and unrelated.

IV. ADJUSTMENTS ON THE OPTUIMUM PARAMETER OF OI-P

A. Optimal Frequencies and Direction Bands for Partitioning

The assimilation procedure is to integrate the model's first-guess and observed partition parameters (e.g., frequency and direction) into an analyzed field of parameters. An important input for the OI-P procedure is the covariance of the errors of the observed and the model parameters. The covariance is obtained by calculating

long-term statistics of differences between the observations and the hind-casts of SWAN model. The observation errors are assumed to be spatially independent.

Although there is only one data buoy in the deep ocean, it is required the first-guess spectra of neighboring grid points of Gagua Ridge buoy be used as fictitious buoys data. Comparing the wave spectral of virtual stations with the field station, the weight between virtual stations and the field station can be acquired. The observed wave spectra data of a 3-month duration from Gagua Ridge buoy are used in carrying out statistic analysis during the process, and then OI-P by these wave spectra can be proceeded. The computer time will be influenced due to the number of directions and frequencies. Therefore, 2-day assimilations in these experiment runs after the model warms up are done to acquire the optimal choices in terms of RMSE (Root Mean Square Error) for comparison on the significant wave height between the observation and simulated data (shown in Table 1). In order to spend the less computer time and get higher accuracy, 16-direction and 20-frequency bands are applied for latter assimilation of typhoon events.

B. Optimizing the Number of Virtual Stations

Generally speaking, it needs two observed stations at least to carry out OI, there is just only one Gagua Ridge buoy station in this study. Therefore, we have to select some stations for optimal interpolation uses Fig. 2 shows the virtual stations on the grid points, which locate near the Gagua Ridge buoy station. Now, three stations, five stations, and seven stations are set up to do numerical tests for finding the appropriate stations. The average errors of SWH and mean wave period (MWP) at Gagua Ridge buoy for differently selected stations for 2-day model simulation are shown in Table 2. It is obvious that the more virtual stations are in the numerical tests, the better the accuracy in the model results. In this study, five virtual stations are established for evaluation.
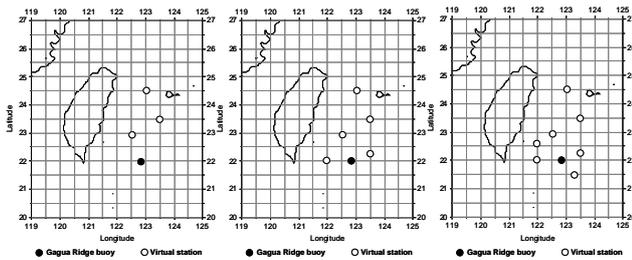
Fig. 2. The virtual stations on the grid points.

TABLE 1
SWH RMSE STATISTICS OF THE VARIOUS
NUMERICAL EXPERIMENTS AGAINST THE DATA
OF GAGUA RIDGE BUOY

Frequency \ Direction	8	16	32
10	0.85	0.73	0.71
20	0.77	0.47	0.43
41	0.58	0.39	0.34

TABLE 2
THE AVERAGE ERRORS OF SIGNIFICANT WAVE
HEIGHTS AND MEAN WAVE PERIODS FOR
DIFFERENT VIRTUAL STATIONS.

Average error	3 virtual stations	5 virtual stations	7 virtual stations
H_s(cm)	18.7	12.8	10.1
T_m(sec)	1.1	0.7	0.4

V. VERIFICATIONS OF THE RESULTS FROM THE ASSIMILATION RUNS AGAINST BUOY OBSERVATIONS

The influence of the assimilation on the wave analyses and forecasts was assessed by running the SWAN wave model for typhoon events, one is for Typhoon Kaemi and the other is for Typhoon Shanshan, in 2006 summer season. In case that the CWB wind fields were missing and then no runs were done, these warm-up periods would be removed from the evaluation.

The effect of OI-P assimilation in the SWAN model are shown in Fig. 3 to Fig. 5 below. In general, these figures show that the results of assimilation runs are much closer to the measurements for the direction wave spectra, one-dimensional spectra, significant wave heights and the mean periods. Fig. 3 shows, for example, the direction wave spectra at Hualien Buoy station on 1600 UTC, July 23, 2006, for buoy observation (Fig. 3a), assimilation run (Fig.3b) and reference run (Fig. 3c), respectively. It is found that the assimilation result is close to the buoy observation for the directional distribution, intensity and main direction of spectra in the rose diagram. For the directional spectral distribution, results of the reference run show a direction shift of 20~30 degrees toward west compared to the observation and assimilation result. Additional high frequency components appear in the reference runs, which are removed by the assimilation.

Fig. 4 shows the one dimensional frequency wave spectra at Hualien buoy; Fig. 4a at July 23 1600UTC for Typhoon Kaemi and Fig.4b (case for September 15, 1500UTC for Typhoon Shanshan. Results also reveal that the same tendency of the wave spectra for both typhoon events exists. The intensity of the wave spectra in the reference runs are lower than in assimilation runs and observations, which are very close. Other features like the second peak around 0.18 Hz in Fig. 4a and 0.15 Hz in Fig.4b are not simulated the reference run.

The SWH time series (Fig. 5) shows the great improvements of the model results by data assimilation for both typhoon events. The hindcast results of SWH in Fig. 5 revealed that neither the peak value nor the timing of peak value are calculated correctly without data assimilation. The oscillations around peak time are modeled very well by the assimilation runs.

The statistics clearly highlight that the assimilation run is much closer to the observations than the reference run. In other words, with the OP-P concept proposed by this study can really enhance the forecast capability, even if only one reference buoy within the forecast domain is used for assimilation. Therefore, it is concluded that the data assimilation has good performance in SWAN wave model simulation for SWH and MWP.

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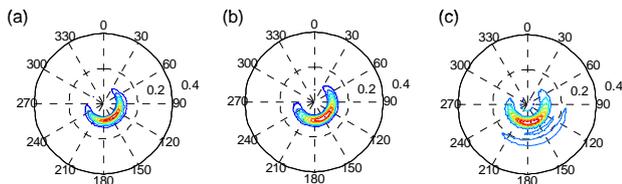
Fig. 3. Direction wave spectra at Hualien buoy for July 23, 2006, 1600UTC. (a) buoy observation, (b) assimilation run, and (c) reference run.
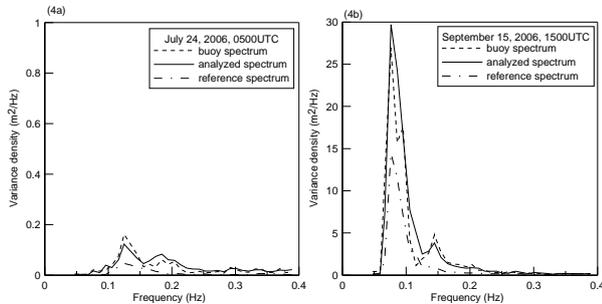
Fig. 4. Spectra at Hualien buoy for July 23, 2006, 1600UTC, and September 15, 2006, 1500UTC.
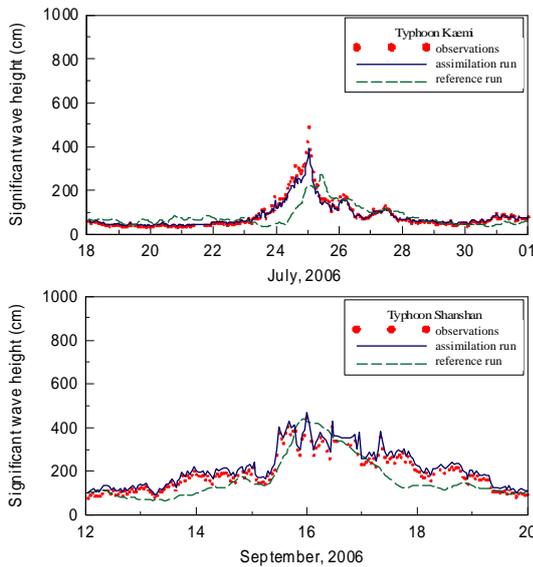
Fig. 5. SWH time series at Hualien buoy. (a) Typhoon Kaemi; (b)Typhoon Shanshan.

VI. CONCLUSIONS AND OUTLOOKS

A spectral wave data assimilation scheme is presented and based on the wave spectrum into separate wave systems and subsequent optimal interpolation of wave partitions. The assimilation experiments in the eastern Taiwan Waters Region show a large improvement on the sea state analysis.

In order to get the optimal number of parameters, numerical results show that the optimal choice for data assimilation lies in 16 directions and 20 frequencies. The number of assumed stations should be greater than 5 stations, and then the errors will tend to be in a stable condition.

This paper also concludes that the assimilation results for both typhoon events are very close to the buoy observations for the directional distribution, the intensity and the mean direction of spectra. Results reveal the same tendency for the wave frequency spectra. The under-prediction of the reference run is clearly corrected by the assimilation. Comparisons of SWH and MWP time series indicate that the performance of model output improve greatly on data assimilation for both typhoon events, Typhoon Kaemi and Shanshan.

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