Application of the Wavelet Transform and Inverse Wavelet Transform to Analyze the Ocean Wave Signals from Data Buoys

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***Abstract*- The ocean wave acceleration signal measured from the data buoy system is a key to derive wave features. It is well-recognized that most of the geophysical quantities are usually non-stationary, so are wave acceleration signals. This study applies a method, based on the theory of the continuous wavelet transform and its inverse, to derive the ocean wave scalogram and sea surface elevation from wave acceleration signals. Irregular wave signals are used to verify the practicality of the wavelet algorithm. After analyzing the natural wave acceleration signals, the wave scalogram presents instant wave nonlinearities at some short-time duration from the wave time series records.**

I. INTRODUCTION

Ocean waves have attracted considerable attentions throughout history. In the present day the mechanism of wave formation and the way that waves travel across the coastal ocean is still not fully understood. Field measurements must be performed to increase practical knowledge of waves. However, most observation sensors are suitable for application nearshore or in shallow water areas. The data buoy is designed to measure wind waves and swell in any depth. A buoy floats on the sea surface and moves up and down with the sea surface. The accelerometer inside the buoy measures the vertical acceleration of sea surface, and the buoy system could obtain heave motion by twice integrating the heave acceleration analytically (Huang and Chen, 1998). Although the concept is simple, there are a number of problems in its successful implementation (Tucker and Pitt, 2001).

Previous studies have often used the Fourier transform (FT) and inverse Fourier transform (IFT) to obtain wave spectrum and sea surface elevation information from the acceleration signals (Dean and Dalrymple, 1991). The FT and IFT assume signals are stationary within the observed duration. However, most of the geophysical quantities are non-stationary, as are the signals from ocean waves. To describe the spectral characteristics of non-stationary signals, the time-frequency map should be applicable. The wavelet method has increased its applications in recent years since its inception in the early 1980s, it is now recognized as a useful, flexible, and efficient technique to analyze intermittent, non-stationary. However, spectral characteristics and sea surface elevation derivation by the wavelet method has received little attention. There are two essentially different approaches in wavelet transform, namely, the continuous wavelet transform (CWT) and the discrete wavelet transform (DWT). The CWT plays the similar role as the Fourier transform and is mostly used for analysis and feature detection in signals, whereas the DWT is more appropriate for data compression and signal reconstruction (Antoine et al., 2004). This article develops a procedure for carrying out the theories of continuous wavelet transform (CWT) and inverse continuous wavelet transform (ICWT) to derive wave spectral information and sea surface level from acceleration records measured from the oceanic data buoys. Simulated and natural wave signals are used for verifying the practicality of this method.

II. THEORETICAL PRELIMINARIES

Based on the theory of the one dimensional CWT, the acceleration signal can be broken into various wavelets, scaled and shifted versions of a pre-chosen mother wavelet function. The time series of acceleration signal $A_c(t)$ corresponds to the acceleration value of each time point t. The continuous wavelet transform $W_{Ac}(b,a)$ of $A_c(t)$ for a transformed mother wavelet is:

$$W_{Ac}(b,a) = G^{-0.5} a^{-0.5} \int \psi_{b,a}(t) \cdot A_c(t)\,\mathrm{d}t \tag{1}$$

$$\psi_{b,a}(t) = a^{-0.5} \psi[(t-b)/a] \tag{2}$$

$$G = \sqrt{\pi/\omega_0^2} \tag{3}$$

in which ω_0 is a constant that forces the admissibility condition (Buessow, 2007), the scaling parameter a is related to the dilated frequency in the time domain. The factor a is a normalization which gives all dilated versions of the mother wavelet the same energy, that is, it is the ratio of the size of the dilated wavelet to the size of the mother wavelet. The translation parameter b corresponds to the wavelet position as

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it shifts through the time domain. $W_{Ac}(b,a)$ conserves the signal norm, thus its total energy:

$$E = \int |A_c(t)|^2 \, \mathrm{d}t = \int\int |W_{Ac}(b,a)|^2 \, \mathrm{d}a \, \mathrm{d}b / a^2 \quad (4)$$

To implement Eq. (1), it is necessary to choose a mother wavelet function ψ first. The Morlet wavelet function, a common wavelet function used in many applications, is chosen here for detecting wave information from the acceleration signal. This study used the Morlet mother wavelet function and its function in the Fourier (spectral) space, as defined in Eqs. (5) and (6), throughout the implementation procedures.

$$\psi(t) = \exp(i\omega_0 t) \cdot \exp(-0.5t^2) \quad (5)$$

$$\hat{\psi}(\omega) = (2\pi)^{-0.5} \cdot \exp\left[-0.5(\omega - \omega_0)^2\right] \quad (6)$$

where $\hat{\psi}$ is the Fourier space of function ψ, it means the function in frequency domain. ω is the frequency. ω_0 was introduced before, Buessow (2007) has discussed the value of ω_0 for the purpose transforming scaling parameter into real frequency ω.

The result of Eq. (1) is called as the scalogram by many studies, the scalogram is a measure of the energy distribution over time shift b and scaling factor a of the signal. Based on the previous section, we can obtain the wave acceleration scalogram $W_{Ac}(b,a)$ from the acceleration signals time series by the CWT method. However, the spectral information is more significant for the applications of oceanography and coastal engineering. Ocean wave scalogram describes the characteristics of sea surface elevation in the time and frequency domains. The techniques to obtain directly the sea surface elevation from the sensor inside the data buoys are still under development at present.

To obtain the wave scalogram by the buoy, the method indirect method to calculate the wave spectrum from the acceleration spectrum can be adopted. In most applications of the spectral concept it is assumed that sea surface elevation can be considered as the linear superposition of a large number of long crested wave trains of low amplitude with different frequencies all traveling independently of one another. The wave spectrum is double integration of the acceleration spectrum. Double integration corresponds to multiplying the wave spectrum by $1/\omega^4$ (Tucker and Pitt, 2001).

III. Verifications

Ocean waves in nature are often random and irregular rather than regular. The algorithm should be developed and confirmed by testing with random waves. The current study applied in-situ wave records to verify practicability of the wavelet algorithm. The sea surface elevation records used here were measured by ultrasonic wave gages on the Cigu pile station from the Taiwan Strait. The station (Fig. 1) is located 3km from the coast of Taiwan; the water depth is about 15 m. This research collected 1500 data from the Cigu pile station and applied the data to verify viability of the wavelet algorithm for sea surface elevation calculation. Figure 2 shows the wave features of the 1500 wave data records. The wave height and wave period conditions of most data are 0.5m~2m and 4sec ~6sec. To verify accuracy of sea surface elevation by the wavelet algorithm on irregular waves, this study calculates the acceleration data by double differential of in-situ sea surface elevation records.

Figure3 shows one case of wavelet scalograms of ocean waves calculated from the sea surface elevation signals and wave acceleration signals. Due to the influence of the transfer function $1/\omega^4$, the wavelet scalogram shows obvious energy concentrated on the very low frequency band. Wang et al. (1993) sees this energy on the very low frequency band as noise. The current work uses a filter with a cut off frequency of 0.03 Hz to eliminate the low frequency noise (Wang et al., 1993). The spectral features between Fig.3(a) and Fig.3(b) are similar. In addition to the wavelet scalogram, this article also discusses the sea surface elevation calculated from the wave acceleration by the algorithms of CWT and ICWT. After applying the wavelet algorithm, the sea surface elevation of irregular wave can be obtained. As Fig. 4 shows, the calculated sea surface elevation fits with the in-situ data record, except for the marginal parts of the wave record.

It's the signal leakage around the marginal areas of the wavelet scalogram. The leakage generated by the limited length of the basic wavelet function. The amount of leakage usually depends on the length of data as well as the decomposition results. In reality, because of the finite data length, even pure sinusoidal components with different frequencies are not exactly orthogonal (Huang et al., 1998). After applying this incomplete wavelet function to the wave record, the energy distribution is biased.

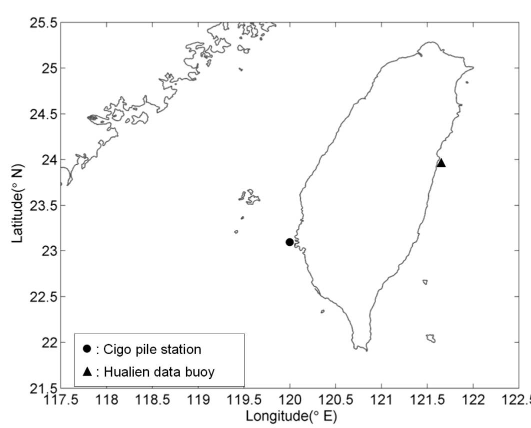

Figure 1. The location of the in-situ stations.

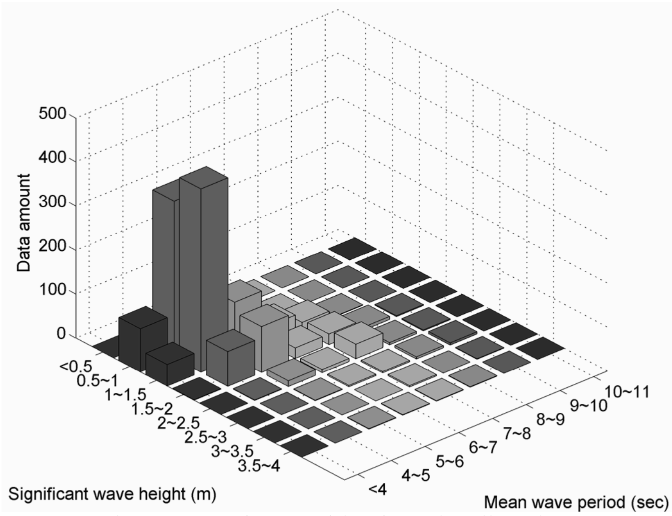

Figure 2. Wave features of data from Cigo pile station.

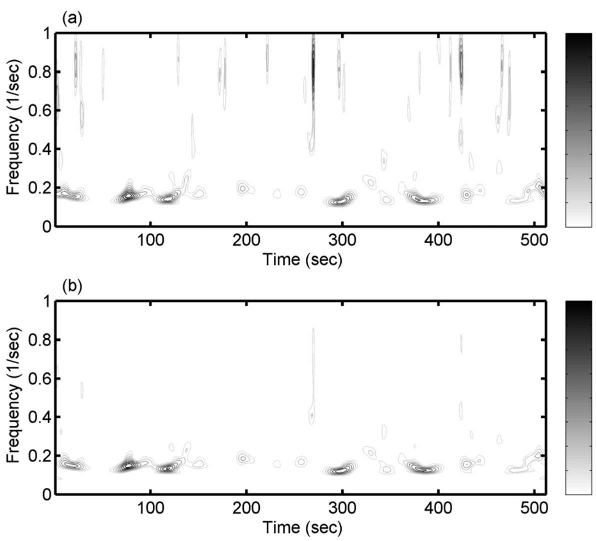

Figure 3. Wave scalograms calculated from the sea surface elevation signals (a) and wave acceleration signals (b).

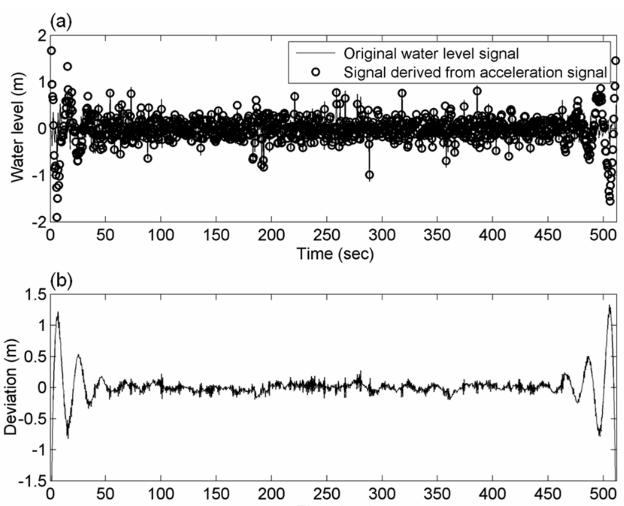

Figure 4. (a) Relationships between the observation sea surface elevation and calculated result by the wavelet algorithm; (b) The deviation of estimated sea surface elevation by the wavelet algorithm.

IV. Wavelet analysis for buoy acceleration data

Here we discuss the features of wave scalogram and sea surface elevation estimated from the buoy acceleration signals by the wavelet and inverse wavelet algorithms. The acceleration signals are collected from the Hualien sea area where is in the eastern part of Taiwan (Fig. 1). This sea area lies on the border between the largest land mass and the largest ocean in the world, so that the marine environments here are complex and sensitive. The acceleration of data buoy motion may not equal to sea surface acceleration exactly, the transfer function which describes the relationship between buoy motion and sea surface motion in frequency domain can effectively correct the spectrum (Huang, 1998). Because this is not the issue of this study, we assume the buoy follows the water surface elevation accurately (Young, 1999).

Figure5 shows one case of wave scalogram and sea surface elevation calculation. This study notices the nonlinear features at short-time duration by observing the scalogram. The energy distribution shows two energy peaks on the 375 sec to 390 sec of the wavelet scalogram. The first peak of the energy distribution locates on 0.1 (1/sec); the second peak locates around 0.2 (1/sec). The similar phenomenon occurs on the 80 sec to 95 sec of the wavelet scalogram as well. It should be the effect of wave nonlinearity. Waves in the ocean are nonlinear, random, and directionally spread, but engineering calculations are almost always made using waves that are either linear and random or nonlinear and regular. By observing the wave spectra from real ocean wave time series, Herbich (1990) pointed out that secondary spectral peak at the frequency about twice the main peak frequency is almost entirely composed of secondary nonlinear components, which belong to the first group of bound waves. The nonlinearity of ocean waves is often conspicuous in coastal regions (Hara and Karachintsev 2003). By observing the wave spectra from ocean wave time series, Herbich (1990) pointed out the secondary spectral peak at the frequency about twice the main peak frequency is almost entirely composed of the secondary nonlinear components, which belong to the first group of bound waves. The same wave data, used in Fig.5, are also analyzed the wave spectrum (Fig. 6). However, the energy at the first harmonic frequency is not obvious. It implies that wave nonlinearity is not obvious for the whole 512 sec time series. Because the instant secondary nonlinear components occur at different frequency bins from different time points of the whole 512 sec time series, the energy at different harmonic frequency bins should be averaged in the FFT spectrum. The wave scalogram reveals that wave nonlinearity is a short-time event within the whole time series. The phenomenon of wave nonlinearity is non-stationary.

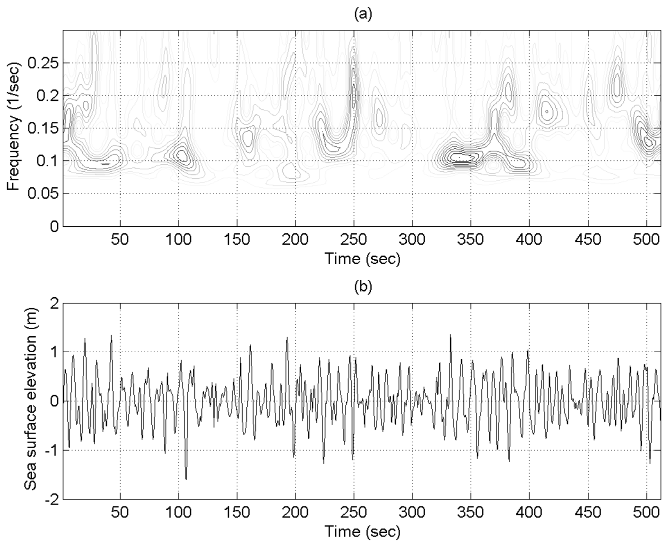

Figure 5. Wave scalogram and sea surface elevation calculation from buoy acceleration signals.

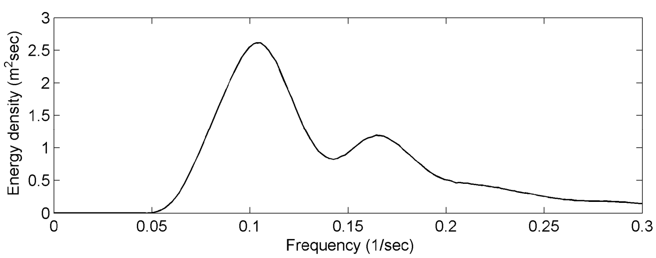

Figure 6. Wave spectrum analysed from the same data set of Fig. 5.

V. CONCLUSIONS

Sea surface elevation and ocean wave spectrum are both significant in marine science research and ocean engineering. The data buoy is a common and popular platform to record wave characteristics in any water depth condition. Because this platform is often floating on the sea surface, the technologies to measure the vertical positions of the sea surface by the buoy system are still under development. Most of the data buoy systems measure sea surface acceleration instead of measuring wave spectrum and sea surface elevation directly. Deriving wave spectrum and sea elevation from the acceleration signal is possible using the applicable algorithm. In case of non-stationarity from the wave signal, this study develops a new technique based on the continuous wavelet transform and inverse wavelet transform to derive the wave scalogram and sea elevation from the acceleration signal. The study collects, defines, and explains required theories of continuous wavelet transform and inverse wavelet transform.

After discussing case of irregular wave signals, this investigation confirms the method for deriving the wave scalogram and sea surface elevation from the acceleration signal using a new wavelet algorithm. Compare with the wave spectrum, the wave scalogram provides the information of wave energy density in both time and frequency domain.

To discuss the wave scalogram and sea surface elevation calculated from natural acceleration signals, this study analyses 1-year acceleration records from buoy accelerometer. The wave scalogram presents instant nonlinear features at short-time duration. However, wave nonlinearity is not obvious from the wave spectra of the whole 512 sec time records. The energy from the spectrum at different harmonic frequency bins should be averaged in the wave spectrum. This is a reason that the energy at the harmonic frequency is not obvious in the wave spectrum but clear in the wave scalogram. Now we could conclude that the superiority of CWT and ICWT on analyzing the instant wave features from the buoy acceleration signals.

ACKNOWLEDGMENT

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