Applying the Wavelet Transform to Derive Sea Surface Elevation from Acceleration Signals

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Abstract- **The current work develops a procedure for carrying out the theories of continuous wavelet transform and inverse wavelet transform to derive sea surface level from acceleration records. Wave signals were applied to verify the practicability of the wavelet algorithm. It is revealed the accuracy of sea surface elevation is influenced by the margins of wave signals, noise in the low frequency band, and the mother wavelet function. After discussing cases of regular and irregular wave signals, this investigation confirms the method for deriving sea surface elevation from the acceleration signal using the wavelet transform.**

I. INTRODUCTION

The ocean wave is a highly complex factor in coastal and ocean applications. The information obtained from ocean waves can potentially help solve problems concerning marine resource conservation and coastal environment protection. The features of ocean waves are extremely random and directly and indirectly depend on meteorological, hydrological, oceanographic and topographical factors. They cannot be fully understood purely by theoretical approaches or through laboratory experiments. Field measurements must be performed to increase the practical knowledge of waves. However, most observation sensors are suitable for application near the shore or in shallow water areas. Apart from remote sensing devices, buoys and vessels are the only platforms to satisfy the wave measurement in deep water areas (Tucker and Pitt, 2001).

The data buoy is designed to measure wind waves and swell in any depth. It is equipped with a tri-axial accelerometer to measure surface wave particle movements for estimating the directional wave spectrum (Doong et al., 2007). A buoy floats on the sea surface and moves up and down with the waves. The sensor inside the buoy measures its vertical acceleration, and the heave motion in a buoy measurement system was obtained by twice integrating the heave acceleration analytically (Huang and Chen, 1998). Although the concept is simple, there are a number of problems in its successful implementation (Tucker and Pitt, 2001). To obtain the wave information from the data buoy, the spectrum was applied. Wave acceleration records measured from data buoy can be transformed into the acceleration spectrum by the suitable spectrum transformation method. Wave spectrum can be obtained from the acceleration spectrum by the transfer function (Capobianco et al., 2002). The distributions of wave energy from the wave spectrum attributed to the movements are then used to gain the precise information in wave heights and periods.

Despite the possibility of obtaining the wave parameters from the spectrum, the record of the sea surface elevation is still necessary for some important topics, such as studies of wave grouping and freak waves (Liu, 2000). It would be more practical if the sea surface elevation information could be obtained from the acceleration record which is measured from the sensor inside the data buoy.

To obtain the sea surface elevation information from the acceleration record, the Fourier and inverse Fourier transform was often used in the past (Dean and Dalrymple, 1991). For the algorithm of the Fourier and inverse Fourier transform, stationarity within the observed period was assumed. However, it is a fact most real signals in nature are non-stationary; as are wave signals. The wavelet transform has increased its applications in recent years since its inception in the early 1980s. It has been applied to solve various engineering problems and almost every part of physics. It is now recognized as a useful, flexible, and efficient technique to analyze non-stationary signals as well as wave records which are obtained from experimental or in-situ measurements (Liu, 2000). However, the issue of sea surface elevation calculation by the wavelet transform has received little attention. Thus, this article develops a procedure for carrying out the theories of continuous wavelet transform and inverse wavelet transform to derive the sea surface level from acceleration records.

II. THEORETICAL PRELIMINARIES

Based on the theory of one dimensional continuous wavelet transform, the acceleration signal can be broken into various wavelets which are scaled and shifted versions of a pre-chosen mother wavelet function. The acceleration signal A_c (t) corresponds to the acceleration value of each time point t .

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The continuous wavelet transform $W_{A_c}(b,a)$ of acceleration signal $A_c(t)$ for a transformed mother wavelet $\psi_{b,a}$ is:

$$W_{A_c}(b,a) = \frac{1}{\sqrt{c_\psi}} \langle \psi_{b,c} | A_c(t) \rangle \quad (1)$$

in which the scaling parameter a is related to the dilated frequency in the time domain. It is a non-dimensional scale factor. The factor a is a normalization which gives all dilated versions of the mother wavelet the same energy, that is, it is the ratio of the size of the dilated wavelet to the size of the mother wavelet. The translation parameter b corresponds to the position of the wavelet as it shifts through the time domain. The function $\psi_{b,c}$ must satisfy two mathematical properties to be classified as 'wavelets'. First, they must have must have finite energy:

$$\|A_c\|^2 = \int_{R^2} |A_c(t)|^2 \, dt < \infty \quad (2)$$

Second, they must have an "admissibility condition", which can be expressed by:

$$C_\psi = (2\pi) \int_{-\infty}^{\infty} \frac{|\acute{\psi}(\omega)|^2}{|\omega|} d\omega < \infty \quad (3)$$

where $\acute{\psi}$ is the Fourier space of function ψ; it means the function in frequency domain. ω is the frequency. Eq. (1) can also be expressed as:

$$W_{A_c}(b,a) = C_\psi^{-1/2} a^{-1/2} \int \psi_{b,a}^{*}(t) \cdot A_c(t) dt \quad (4)$$

$$\psi_{b.a}(t) = \frac{1}{\sqrt{a}} \psi\left[\frac{i-b}{a}\right] \quad (5)$$

where ψ^* is the complex conjugate of the wavelet function ψ. $W_{A_c}(b,a)$ conserves the norm of the signal, thus its totalenergy (Buessow, 2007):

$$E_n = \int |A_c(t)| dt = \iint |W_{A_c}(b,a)|^2 \frac{da db}{a^2} \quad (6)$$

To implement Eq. (5), it is necessary to choose a mother wavelet function ϕ first. The Morlet wavelet function, which is a common wavelet function used in many applications, is chosen here for detecting the wave information from the acceleration signal. The Morlet mother wavelet function and its function in the Fourier (spectral) space, as defined in Eqs. (7) and (8), were used throughout the implementation procedures in this study.

$$\psi(t) = e^{i\omega_0 t} \cdot e^{(-0.5t^2)} \quad (7)$$

$$\psi(t) = e^{(i\omega_0 t)} \cdot e^{-0.5(\omega-\omega_0)^2} \quad (8)$$

where ω_0 is a constant that forces the admissibility condition, as shown in Eq. (3), to be satisfied. It was suggested to set up as 5.5 following the previous study (Jordan et al., 1997). For calculating the sea surface elevation time series $\eta(t)$ from the energy, the inverse wavelet transform is applied here:

$$\eta(t) = \frac{1}{C_\psi} \int_{-\infty}^{\infty} \int_{-}^{\infty} W_\eta(b,a) \cdot \psi_{b,a}(t) db \frac{da}{a^2} \quad (9)$$

where $W_\eta(b,a)$ is the continuous wavelet transform of the sea surface elevation time series $\eta(t)$, it is related to $W_{A_c}(b,a)$.

A regular wave in the time domain, the water level could be given by:

$$\eta(t) = P \cdot \cos(2\pi f t + \varepsilon) \quad (10)$$

in which P is the wave amplitude, f the frequency, and ε the phase of the wave component. The sea surface elevation is the double integral of the wave acceleration signal $A_c(t)$. In other words, $A_c(t)$ is the double differential of $\eta(t)$:

$$A_c(t) = \left(\eta(t)\frac{1}{dt}\right)\frac{1}{dt} = -(2\pi f)^2 P \cdot \cos(2\pi f t + \varepsilon) = -(2\pi f)^2 \eta t \quad (11)$$

As shown in Eq. (11), the transfer function between the wave acceleration and water level function should be $-(2\pi f)^2$:

$$\eta(t) = -(2\pi f)^2 A_c(t) \quad (12)$$

As shown in Eq. (4), the relationship between $W_{A_c}(b,a)$and $A_c(t)$ has been revealed. Similar to Eq. (4), the relationship between $W_\eta(b,a)$and $\eta(t)$ was shown here:

$$W_\eta(a,b) = C_\psi^{-1/2} a^{-1/2} \int \psi_{(b,a)}^{*}(t) \cdot \eta(t) dt \quad (13)$$

Combining Eqs. (4), (12) and (13), we can calculate the wavelet coefficient of the sea surface elevation from thewavelet coefficient of the wave acceleration signals:

$$W_\eta(b,a) = -(2\pi f)^{-2} \cdot W_{A_c}(b,a) \quad (14)$$

Therefore, the sea surface elevation can be obtained from the wave acceleration records by Eqs. (9) and (14). The wavelet scalogram is called $|W_\eta(,a)|^2$ (Yeh and Liu, 2008), the scalogram is a measure of the energy distribution over time shift b and scaling factor a of the signal. According to the law of the conservation of energy, the relationship between the water level and wavelet scalogram is shown here:

$$E = \int_{-\infty}^{\infty} |\eta(t)|^2 dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} \left|W_\eta(b,a)\right|^2 \frac{dadb}{a^2} \qquad (15)$$

The transfer function between the wavelet scalogram $\left|W_\eta(b,a)\right|^2$ and $\left|W_{A_c}(b,a)\right|^2$ is shown here:

$$\left|W_\eta(b,a)\right|^2 = (2\pi f)^{-4} \cdot \left|W_{A_c}(b,a)\right|^2 \qquad (16)$$

III. VERIFICATIONS

To understand the feasibility of deriving the sea surface elevation from the non-stationary acceleration signal, we simulated the wave signals in the time domain:

$$\eta(t) = \begin{cases} P_1 \cdot \cos(2\pi f_1 t) & t \le T/2 \\ P_2 \cdot \cos(2\pi f_2 t) & t \le T/2 \end{cases} \qquad (17)$$

in which T is the total time duration of the sea surface elevation, the definitions of other parameters are the same as those in Eq. (10). In Eq. (17), P_1= 1 m, f_1 = 0.1 Hz, P_2 = 0.5 m and f_2 = 0.2 Hz are used to simulate two different kinds of wave system. To analyze discretely sampled data, such as the wave records from the wave sensor, it is necessary to sample the sea surface elevation function. A simplified example as shown in Fig. 1 and Eq. (18) could be introduced to explain the idea:

$$T = N_t \Delta t \qquad (18)$$

The discrete velocity and acceleration of the sea surface elevation can be obtained by:

$$V_e(m_t) = \frac{[\eta(m_t) - \eta(m_t - 1)]}{\Delta t} \qquad m_t = 2,..,N_t \qquad (19)$$
$$A_c(n_t) = \frac{[V_e(n_t) - V_e(n_t - 1)]}{\Delta t} \qquad n_t = 2,....,N_t - 1 \qquad (20)$$

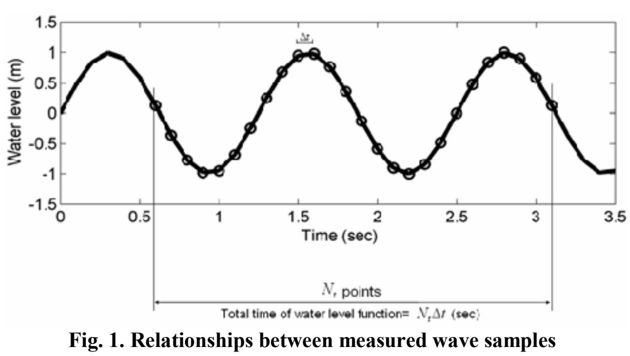


Fig. 1. Relationships between measured wave samples and the continuous wave function.

For this study, the sampling rate of wave data, both from simulation and in-situ observed data, are 2 Hz. This means Δt = 0.5 sec in the cases of study. The data samples are 1024 points for every case of the study. In other words, the duration of wave data records would be 512 sec. Fig. 2(a) shows the wavelet scalogram of the wave signals calculated by the theories of wavelet transform and Eq. (14) from the simulated water level signal. Because of the capability for time-frequency analysis, the scalogram presented the energy of the signals distributed over the time-frequency domain. The spectral characteristics in different time points from the whole time domain are presented by the wavelet scalogram. As shown in Fig. 2(a), obvious energy is concentrated in the very low frequency band. This is influenced by the transfer function (2）2− πf− which was described in Eq. (14). This energy on the very low frequency band is seen as noise; is necessary to remove the energy from the very low frequency band from the wavelet scalogram. A filter with a cut off frequency of 0.03 Hz was used to eliminate the low frequency noise (Wang et al., 1993). Fig. 2 (b) shows the wavelet scalogram result after removing the noise from the low frequency band. The energy distribution from Fig. 2 (b) is concentrated on the frequency bands of 0.1 Hz during the first half of the scalogram in the time domain, and is concentrated on the frequency bands of 0.2 Hz during the latter half of the scalogram. This agrees with the input conditions of the simulated wave signals.

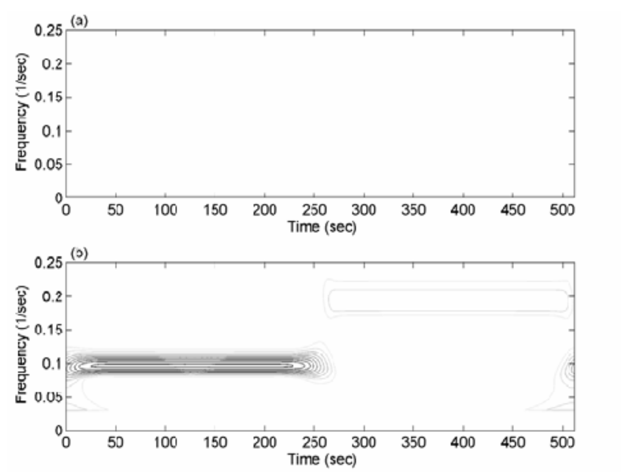


Fig. 2. (a). The wavelet scalogram of wave signals which are calculated by the wavelet transform and Eq. (14); (b). The wavelet scalogram result after eliminating the noise from the low frequency band.

The result of calculated sea surface elevation from the wave acceleration records by the wavelet transform is shown in Fig. 3. The relative differences between the theoretical values and the wavelet results were less, except for the locations near the margins of the wave record. These differences might come from the algorithm, in which we discretized the continuous wavelet transform for analyzing the digital wave records (Fig. 1). This kind of error should be improved by increasing the sampling rate of the wave data. In addition, clear errors occur in the marginal areas because of the biased energy in the marginal area of the scalogram.

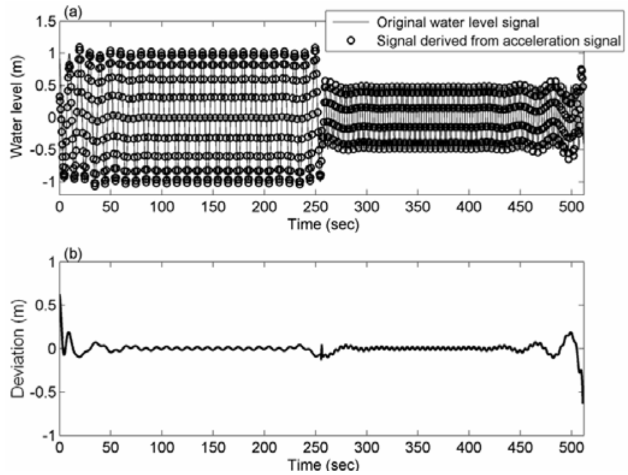
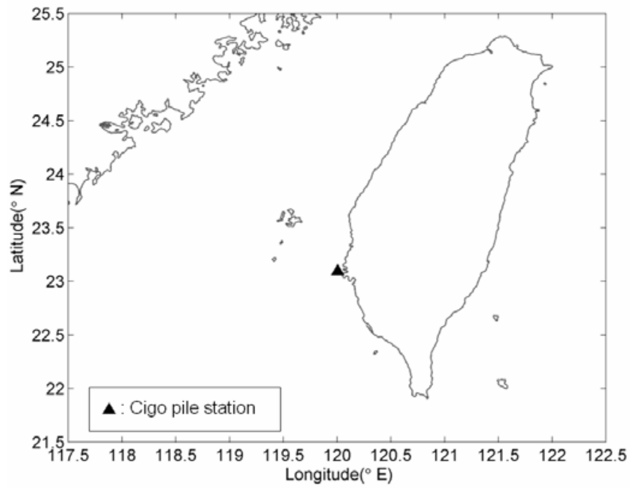


Fig. 3. The case of the regular water level signal: (a) Relationships between the original water level signal and calculated result by the wavelet algorithm; (b) The deviation of the estimated water level by the wavelet algorithm.

Ocean waves in nature are often random and irregular rather than regular. The algorithm for derived water level information from the acceleration should be developed and confirmed by testing with random waves. The in-situ wave records were applied to verify the practicability of the wavelet algorithm. The sea surface elevation records used here were measured by the ultrasonic wave gages on the Cigu pile station from the Taiwan Strait. The station (Fig. 4) is located 3km from the coast of Taiwan; the water depth there is 15 m. 1500 data collected from Cigu pile station are applied to verify the viability of the wavelet algorithm for sea surface elevation calculation. The wave features of the 1500 wave data records are shown in Fig.5. The wave height and wave period conditions of most data are 0.5m~2m and 4sec ~6sec.

To verify the accuracy of sea surface elevation by the wavelet algorithm on the cases of irregular waves, we calculate the acceleration data from in-situ sea surface elevation records using Eqs. (19) and (20). After applying the wavelet algorithm, the sea surface elevation of irregular wave can be obtained. As shown in Fig. 6, the calculated sea surface elevation fits with the in-situ data record, except for the marginal parts of the wave record. The influence of the margin upon the water level derivation will be discussed later.



Figu. 4. The location of Cigo pile station.

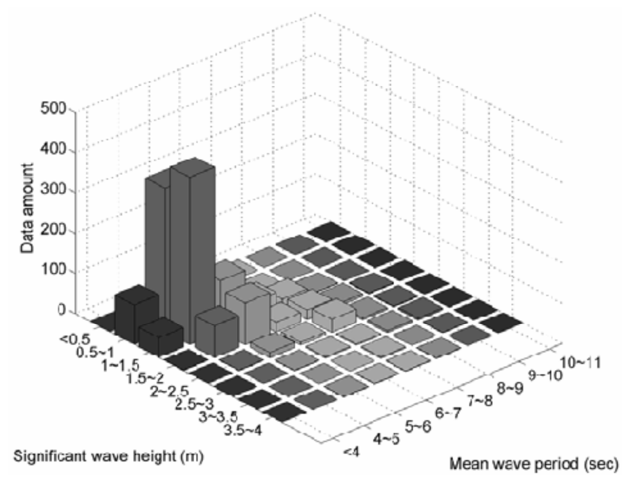


Fig. 5. The wave features of data from Cigo pile station.

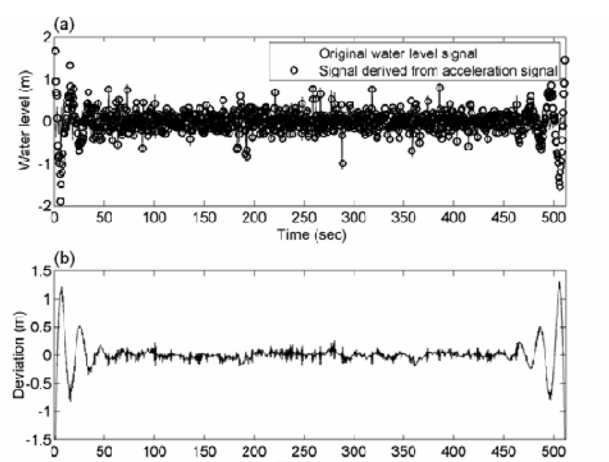


Fig. 6. The case of an irregular wave signal: (a) Relationships between the observation water level and calculated result by the wavelet algorithm; (b) The deviation of estimated water level by the wavelet algorithm.

IV. SUMMARY

Sea surface elevation is important information in ocean engineering and marine science research. However, it is quite difficult to measure in deep water because the platform is often floating on the surface. The data buoy is a common and popular platform for measuring sea surface acceleration in any water depth condition. Deriving sea elevation from the acceleration signal is possible using the applicable algorithm.

Previous studies highlight that spectrum transformation and inverse spectrum transformation are key processes to calculate sea elevation. In case of non-stationarity from the wave signal, this study develops a new technique based on the continuous wavelet transform and inverse wavelet transform to derive sea elevation from the acceleration signal. The study collects, defines, and explains required theories of continuous wavelet transform and inverse wavelet transform.

After discussing cases of regular and irregular wave signals, this investigation confirms the method for deriving sea surface elevation from the acceleration signal using the wavelet transform.

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