

Nonlinear influences on ocean waves observed by X-band radar

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Abstract The purpose of this study is to discuss the influence of signal nonlinearity upon X-band radar observations. A method for estimating the degree of nonlinearity by bispectral analysis was applied and discussed. We found that bispectral analyses from spatial radar backscatter series are similar to results obtained from water level time series. In addition, the average nonlinear degree from radar backscatter is related to wind speed. The accuracy of wave observations derived by consideration of the nonlinear effect from radar backscatter was also investigated. The estimated error in wave height from the radar data is also related to the degree of nonlinearity. In order to improve accuracy, the modulation transfer function method was applied in order to eliminate the influence of nonlinearity.

Keywords Radar backscatter · Nonlinearity · Bispectral analysis

Introduction

Ocean waves have attracted the attention of both the general public and scientists throughout history. In the present day, understanding of the mechanisms of wave formation and the way in which waves travel across the ocean is still not complete. Knowledge of wave characteristics is important in modern coastal technology and oceanographic studies. Wave measurement is a useful way to understand and describe wave characteristics. This process can be largely classified into two categories as in situ and remote sensing. While in situ instruments are used to reconstruct time variations of waves at a single point, remote sensing techniques give information over a broader area. Because waves are distributed over a large-scale region in the spatial domain, the spatial characteristics of ocean waves should also be studied in greater detail. Several studies about ocean waves using remote sensing have been performed since 1960s (Pidgeon 1968; Valenzuela and Laing 1970; Alpers and Rufenach 1981; Lee et al. 1996), and satellite images have often been used for detecting and studying waves. However, because most of the satellites travel along a predetermined track, it is difficult to track the same wave continuously. New technologies for obtaining continuous images of waves have been proposed since the 1980s. It is possible to monitor the same sea area continuously for an operational purpose. In addition, spatio-temporal wave information can be acquired from continuous X-band radar imaging, which should be a potential tool for wave observations. Using the marine X-band radar, it is possible to obtain directional wave spectrum, wave height, wave period, and wave direction (Young et al. 1985; Borge and Soares 2000). The measurement of ocean waves by X-band radar is based on spatial and temporal structural analysis. To obtain wave field information in space and

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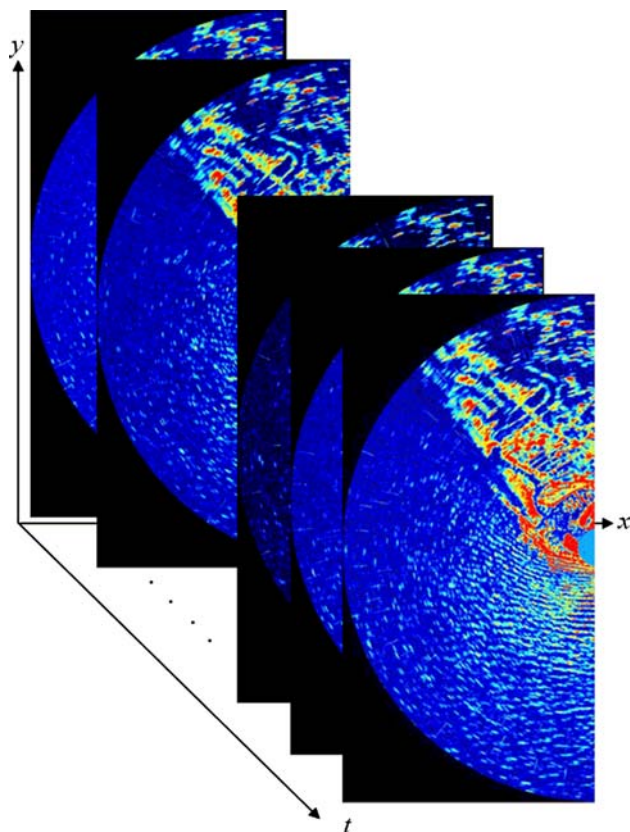


Fig. 1 X-band continuous radar image sequences. They provide the wave field information in space and time domain

time domain, continuous radar image sequences are necessary (Fig. 1). Because radar backscatter is generated by the interaction of electromagnetic waves with the sea surface ripples, sea states can be retrieved from radar image sequences that are composed of radar backscatter (Borge et al. 1999). The first step in estimating the wave parameters from radar images is to calculate the radar image sequences spectrum. An image sequence spectrum is the result of spectral transformation from image sequences. After the spectral transformation, the time (t) and space domain (x, y) functions are transformed to the frequency (f) and spatial frequency (k_x, k_y) domain. A three-dimensional Fourier transformation is used as a tool for transforming the spatio-temporal information into the spectral wavenumber-frequency domain (Senet et al. 2001). In the field of oceanography, spatial frequency represents wavenumber. Accordingly, determining how to obtain the correct radar image sequences spectrum is a key point in retrieving wave information accurately from radar backscatter data.

While analyzing the image sequences spectrum, the energy associated with ocean waves can be separated from background noise by applying wave dispersion relationships as a filter (Young et al. 1985). For linear wave theory, ocean waves are dispersive and show a defined relationship

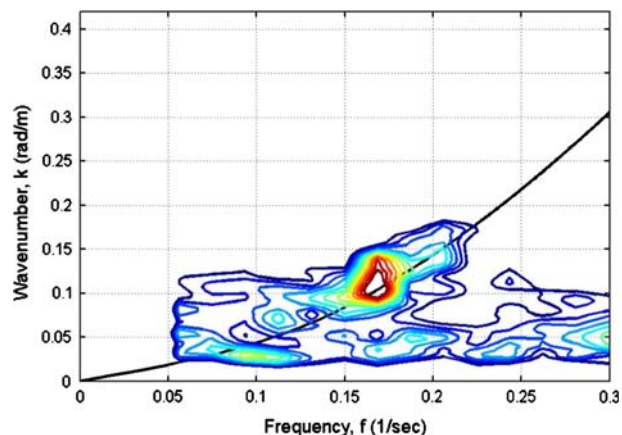


Fig. 2 Sequences of Radar image spectrum. Linear wave dispersion relation is described as the curve. The energy of image sequences spectrum is described as the contour. It can be found that not all of energy matches with the dispersion relation in both frequency and wavenumber domains

between wavenumber and frequency. The dispersion relation is defined as

$$\omega = \sqrt{gk \tanh(kd)} + \bar{k} \cdot \bar{U} \quad (1)$$

In Eq. 1, d is the water depth, \bar{k} is the wavenumber vector and \bar{U} is the surface current vector. The dispersion relationship can be described in the wavenumber-frequency space from an image sequences spectrum. The dispersion relation curve is shown in Fig. 2. The energy of ocean waves is located in the vicinity of the dispersion curve. The wave energy of the radar image sequences spectrum is the summation of energy that is located at the frequency band, where the linear ocean waves belong. The noise from the radar image sequences spectrum is thus the difference between the total energy and the ocean wave energy derived from the image sequences spectrum. The estimated significant wave height is thus based on a linear correlation with the ratio of signal energy to noise energy from the radar image sequences spectrum (Borge et al. 1999).

The present method for estimating the wave parameters from radar backscatter is based on the relationship between spectral energy and noise, which are both estimated by the dispersion relation of linear wave theory. However, the features seen in the radar backscatter data are nonlinear. Radar backscatter would result in shadowing if the higher waves hide the lower waves from radar antenna illumination, such as in the case of lower grazing incidence (Borge and Soares 2000). Because the higher waves hide the lower waves at low grazing incidence, the radar backscatter intensity drops sharply in this scenario. The waveform of radar backscatter would not be very smooth and symmetrical, as in the waveform of a sine or cosine wave, but

would instead be skew and asymmetrical. Shadowing is a non-linear phenomenon that introduces additional components of frequency and wavenumber (spatial frequency) domain into the radar image sequences spectrum (Nieto and Guedes 2000). In addition, the nonlinearity of ocean waves is often conspicuous in coastal regions (Hara and Karachintsev 2003); radar backscatter that has interacted with ocean waves should also be nonlinear. This phenomenon is shown in Fig. 2, which shows that some energy lies off the dispersion relation curve. In addition to the influence of shadowing, there are some factors that attribute to this phenomenon; e.g. the influence of background noise. Cloud, raindrops, and dust are all sources of background noise in radar observation (Long 2001). Some studies have tried to apply filters to eliminate background noise from remote sensing images (Watson 1993; Granville and Rasson 1993). However, because the source of background noise is very complicated, it is almost impossible to eliminate background noise completely.

If a signal series $\{X_t\}$ has the linear representation, it can be expressed as Eq. 2 (Rao and Gabr 1980). In this, $\{e(t)\}$ is a sequence of independent, identically distributed random variables with $E\{e(t)\} = 0$, $E\{e^2(t)\} = \sigma^2$.

$$X_t = \sum_{u=-\infty}^{\infty} a(u)e(t-u) \quad (2)$$

If the signal series is nonlinear, Eq. 2 will be not true. For power spectrum analysis, it is supposed that the signal is combined linearly by different kinds of sine waves. As a result, the power spectrum may not describe the real characteristics of nonlinear signals. Earlier workers have shown the influences of nonlinearity on radar backscatter power spectra. Senet et al. (2001) proposed that the nonlinearity of a radar signal would affect the energy distribution in the domain of radar image sequences spectrum, causing the distribution of ocean wave energy against the dispersion relationship. In other words, the energy distribution of an image sequences spectrum would not match the dispersion relation curve shown in Fig. 2. Wolf and Bell (2001) studied the characteristics of X-band radar image sequences spectra and found that the energy from does not always correspond to the linear dispersion relation curve, especially in the lower and upper frequency band. Therefore, we chose to explore the nonlinear features of radar backscatter in its application to sea states monitoring.

Since the 1960s, researchers have pointed out the usefulness of higher order spectra in analyzing nonlinear time series (Hasselmann et al. 1963; Rao and Gabr 1980; Elgar and Guza 1985). Because of the complexity of higher order spectra, most research has focused on the second order spectrum, which is termed the bispectrum.

The purpose of this study is to discuss the accuracy of X-band radar in wave observation by considering the influence of signal nonlinearity. In order to understand the degree of nonlinearity of every radar datum, the method for estimating it from the bispectrum is applied and discussed. The bispectra of different areas from the radar image are analyzed in order to investigate the nonlinear features of radar backscatter series in the spatial domain. The relationship between meteorological factors and the degree of nonlinearity of the radar backscatter is also highlighted. Finally, we investigate the influence of nonlinearity upon the wave height accuracy by radar observation, and try to improve the estimation of wave height accuracy by considering nonlinear characteristics.

Methodology

Most of the signals in nature are nonlinear. The method for investigating nonlinear signal by bispectrum is described here. Let $X(i)$ be the discrete radar backscatter, which is a one-dimensional series in space domain. The moments of $X(i)$ are defined as follows.

$$M'(0) = \mu = E\{X(i)\} \quad (3)$$

$$M''(0) - [M'(0)]^2 = \sigma^2 = E\{X(i) - \mu\}^2 \quad (4)$$

$$R(n) = \text{cov}\{X(i), X(i+n)\} \quad (5)$$

$$M(n_1, n_2) = E\{[X(i) - \mu][X(i+n_1) - \mu][X(i+n_2) - \mu]\} \quad (6)$$

$E\{x\}$ denotes the expectation value of x , and $\text{cov}\{x_i, x_j\}$ denotes the covariance of x_i and x_j . Because $X(i)$ is a real valued process, the first moment $R(n)$ is symmetric.

$$R(-n) = R(n) \quad (7)$$

$$M(n_1, n_2) = M(n_2, n_1) \quad (8)$$

Assuming the existence of Fourier transforms of $R(n)$ and $M(n_1, n_2)$, the spectral density function $f(k)$ and the bispectral density function $B(k_1, k_2)$ can be defined as Eqs. 9 and 10 (Rao and Gabr 1980). Because the radar backscatter series belongs to spatial information, the independent variables of bispectrum (k_1, k_2) have both spatial frequencies, which is termed the wavenumber.

$$f(k) = \sum_{n_1=-\infty}^{\infty} R(n) e^{-ikn} \quad (9)$$

$$B(k_1, k_2) = \sum_{n_1=-\infty}^{\infty} \sum_{n_2=-\infty}^{\infty} M(n_1, n_2) e^{-i(k_1 n_1 + k_2 n_2)} \quad (10)$$

If the energy of bispectrum is zero in every non-zero spatial frequency domain, then the signal can be defined as a Gaussian process and linearity. In practice, this is almost impossible for a signal in the real world. If the energy of bispectrum is not zero in the non-zero spatial frequency (k_1, k_2) , this means that the nonlinear interaction is caused by the spatial frequencies k_1 and k_2 .

As a result of the effect of shadowing, as mentioned above, the spectral peak at the spatial frequency k_0 is termed the fundamental mode and the peaks at integer multiples of k_0 are called the harmonic modes of the spectrum (Seemann et al. 1997). In this study, we focus on the nonlinear characteristics that are related to harmonic modes. Hence, the nonlinear degree is calculated from bispectra by summing the energy at every spatial frequency (k_1, k_2) where $k_1 = k_2$.

Bispectral analysis is a well-known method for analyzing the nonlinear characteristics of different signals. It was also applied in the field of oceanography. In this study, the method of bispectral analysis is applied in order to determine the nonlinear characteristics from radar backscatter series. Because the bispectrum belongs to the second order spectrum, we must assume that the nonlinearity of radar backscatter is only affected by two components k_1 and k_2 .

Radar data analysis

Data source

The layout of our field experiment is shown in Fig. 3. The area chosen for radar wave observation is in the northeastern Taiwan offshore, where the water depth range is from 30 to 100 m. The location of the radar antenna is about 20 m above the average water level. For the sake of discussing the nonlinear features from different radar data, the radar backscatter series from different radar images are acquired. As shown in Eqs. 6 and 10, the bispectrum of two-dimensional radar image is a four-dimensional dataset. It is quite difficult to explain and present the nonlinear phenomena from four-dimensional data. As a result, the space series of radar backscatter was used instead of the whole two-dimensional radar images. Figure 4 shows one of the radar images used in our experiment. We acquired radar backscatter series between (A) and (B) (Fig. 4), and analyzed about 1500 samples. In

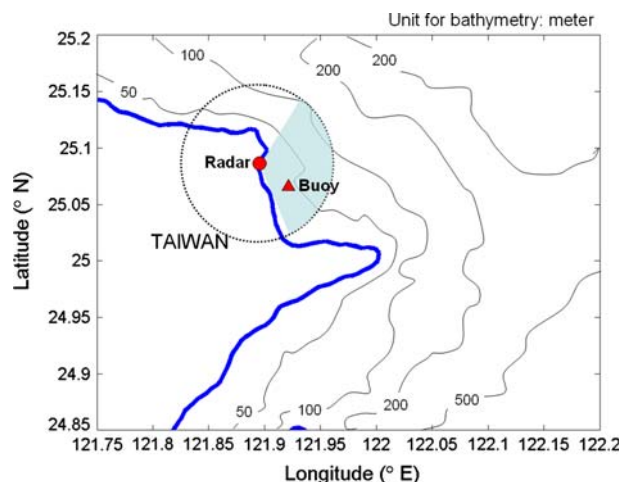


Fig. 3 Layout of radar experiment. The range of water depth for this experiment is from 30 to 100 m. A data buoy is set up in the radar observing area. The measurement results from the data buoy would be applied for comparing with the radar results

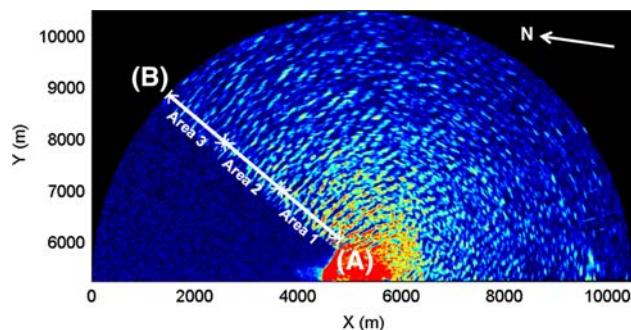


Fig. 4 X-band radar clutter image. In order to detect the degree of nonlinearity from the radar image, the radar backscatter series are selected from (A) to (B). The wave direction is close to northeast in this case

order to discuss the relationship between nonlinear degree and in situ environmental factors, the meteorological and marine data from in situ buoy are used to compare with the radar results.

The bispectrum and nonlinear degree from radar backscatter

Elgar and Guza (1985) verified that nonlinear features could be detected easily in shallow water using bispectral analysis of water level time series. The nonlinear features seen in the radar backscatter series may not depend completely on the characteristics of ocean waves. The bispectral features from different areas in the spatial domain were investigated here. The radar backscatter series are selected from radar images. The selected radar backscatter series are shown in Fig. 5; three sub-series of radar

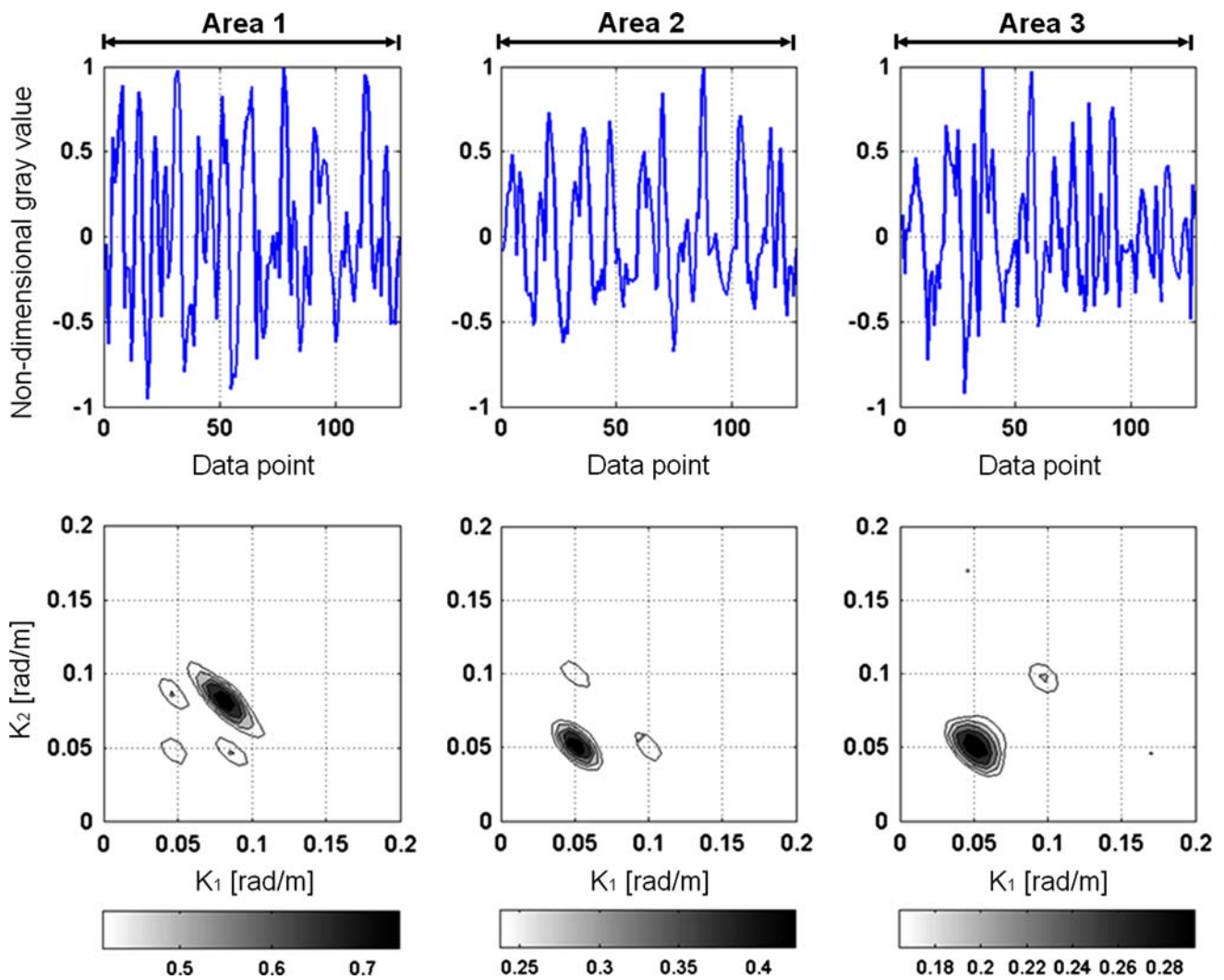


Fig. 5 The bispectrum of backscatter series from different areas in radar images. The nonlinear features in different areas are almost independent. From left to right, the upper figures represent the radar

backscatter from Areas 1 to 3; the harmonic energy peak locates in a higher spatial frequency (k_1 and k_2) domain in the case of the shallow water area (Area 1) than for the deeper water area (Areas 2 and 3)

backscatter were selected from Areas 1, 2 and 3. The corresponding water depth is shown in Fig. 6. Each backscatter series has a zero mean and is normalized individually for decreasing the influences on the bispectrum. The bispectra from different backscatter series were analyzed and the results are shown in Fig. 5. The harmonic energy peak is located in a higher spatial frequency (k_1 and k_2) domain in the case of shallow water area (Area 1) than for the deeper water area (Areas 2 and 3). In addition, the harmonic energy peak in the shallow water area is higher than that in the deeper water area. This means that the degree of nonlinearity in shallow water is higher than in deeper water. This result is similar to the bispectral results derived from analyzing water level time series, because the degree of nonlinearity of ocean waves would theoretically be expected to be stronger in shallow water than in deep water.

The relationships between nonlinear degree and wind speed

Radar backscatter characteristics would also be affected by the properties of wind, especially for grazing angles under

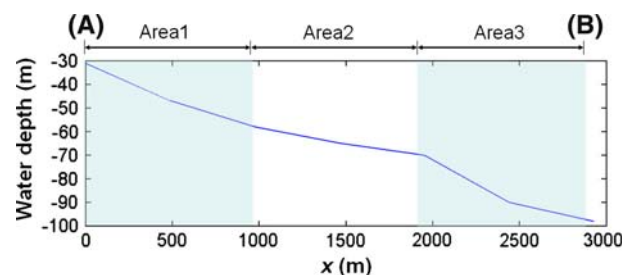


Fig. 6 The variation of water depth from (A) to (B) in Fig. 4. The shallowest water depth in the area is about 30 m

50° (De Loor 1983). Because the grazing angle of radar observation in the present experiment is less than 50°, its influence needs to be accounted for. Figure 7 shows the relationships between the degree of nonlinearity and the wind speed. Wind speed is measured simultaneously by an in situ data buoy, whose location is shown in Fig. 3. In order to understand the relationship quickly, the wind speed data are classified by the Beaufort scale.

The relationship is described by a box-and-whisker plot (Fig. 7). The box itself contains the upper and lower edges of the box are defined to be 75th and 25th percentile of the data set. The upper and lower whiskers are defined as the maximum and minimum of the data set. Unlike most box-and-whisker plots, the horizontal lines in the boxes are defined as the average of the data set in this study, in order to understand the characteristics of averaged nonlinear degree in each Beaufort scale.

Figure 7 shows little correlation between the degree of nonlinearity and wind speed. However, the average degree of nonlinearity of radar backscatter is related to wind speed. The higher the wind speed observed, the higher the average degree of nonlinearity detected from the radar data. Our results verify the studies of De Loor (1983) by using the degree of nonlinearity applied in our study.

The relationships between nonlinear degree and wave accuracy

In order to estimate wave height (H_s) from X-band radar image sequences, the method of Borge et al. (1999) is applied here.

$$H_s = A + B\sqrt{\text{SNR}} \quad (11)$$

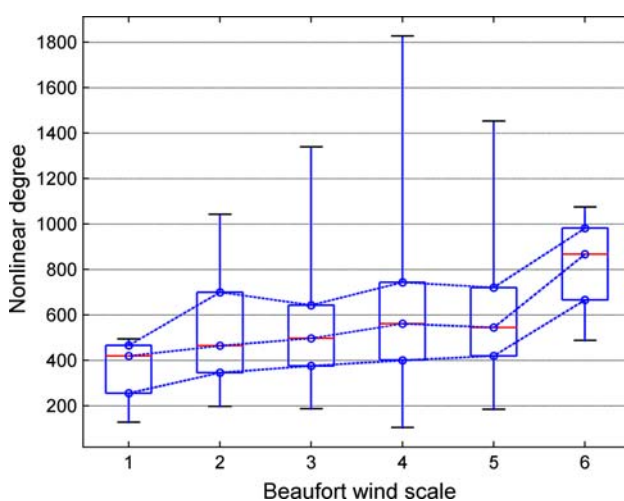


Fig. 7 The relationship between Beaufort wind scale and nonlinear degree of radar backscatter. The average degree of nonlinearity increases with Beaufort wind scale

where A and B are calibration constants. Signal-to-Noise ratio (SNR) is defined by the ratio of wave signal and background noise from image sequences, and is described in Eq. 12:

$$\text{SNR} = \frac{\int F(\bar{k}_s, \omega_s) d\bar{k}_s d\omega_s}{\int F(\bar{k}_b, \omega_b) d\bar{k}_b d\omega_b} \quad (12)$$

where $F(\bar{k}_s, \omega_s)$ is the energy density, \bar{k}_s and ω_s , respectively, are the wavenumber and angular frequency of the linear wave dispersion relation. $F(\bar{k}_b, \omega_b)$ expresses the energy density of the components \bar{k}_b and ω_b which does not obey wave dispersion relation (Borge and Soares 2000). The dispersion relation was described in Eq. 1. The energy of ocean waves $F(\bar{k}_s, \omega_s)$ should be located in the vicinity of the dispersion curve, which is shown in Fig. 2. The noise $F(\bar{k}_b, \omega_b)$ is thus the difference between the total energy and the ocean wave energy derived from the image sequences spectrum. Because the estimated wave height from the radar data is based on the linear wave dispersion relationship, we speculate that the estimated accuracy of the wave height may be influenced by the degree of nonlinearity in the radar backscatter. As a result, we discuss the relationship of the wave height accuracy derived from the radar observation and the degree of nonlinearity calculated from the radar backscatter data.

About 1,500 samples from the same area were analyzed in order to determine the influence that the degree of nonlinearity has on estimating wave height. The wave height data from data buoy, which is taken as the ground truth, is applied in order to understand the accuracy of radar observations. The wave height time series from our data sample is shown in Fig. 8. The range of wave height in our sample is from 30 to 430 cm. The box-and-whisker plot for the degree of nonlinearity and the deviation between radar and data buoy observations is shown in Fig. 9. The horizontal lines in the boxes are also defined as the average of the data set in this study. The deviation of wave height is defined as the absolute deviation between the observed wave height derived from radar and data buoy observations. We found that the deviation in wave height is clear if the degree of nonlinearity is large. In addition, the maximal deviation of every box-and-whisker plot is also related to the degree of nonlinearity. The higher the degree of nonlinearity detected, the larger the deviation distributed in the box-and-whisker plot. This result suggests a way to improve the estimated wave height accuracy in the case of higher degrees of nonlinearity by considering the nonlinearity.

According to the results shown in Fig. 9, it is noteworthy that the maximum error in estimating the wave height is clear if the degree of nonlinearity is large. The wave height was estimated by the signal-to-noise ratio

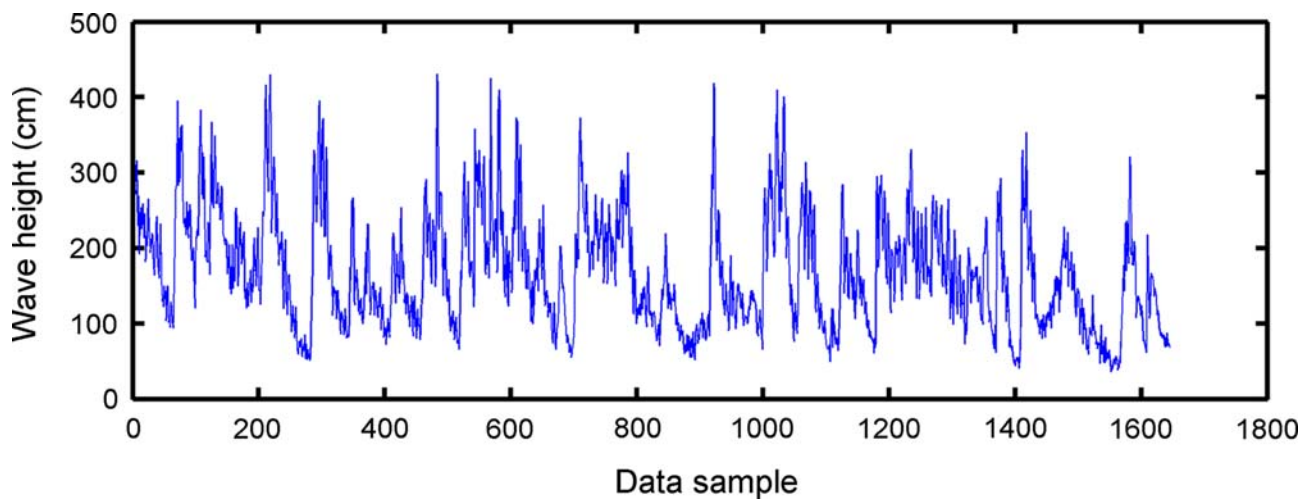


Fig. 8 The wave height time series from our data sample

method (Borge et al. 1999) as already discussed. The estimated accuracy of the wave height may be improved by considering nonlinearity. In order to eliminate the influence of nonlinearity, we applied the modulation transfer function (Borge et al. 2004) as a nonlinear transform method to modify the radar image spectrum. We call it as the modulation transfer function method, and it is applied in this work. The modified result is shown in Fig. 10. In harmony with the definition of the box-and-whisker plot in Fig. 9, the upper whiskers in Fig. 10 show the maximum deviations of estimated wave height between the radar and in situ data buoy observation. In comparison with the results shown in Fig. 9, the

maximum deviations in Fig. 10 are almost constant; they are all limited in 130 cm for the cases of different kinds of nonlinear degrees. Our analysis shows that the maximum deviation of estimated wave height decreases in the case of larger degrees of nonlinearity (nonlinear degree >500) using the modulation transfer function method instead of the signal-to-noise ratio method. In the case of smaller degrees of nonlinearity (nonlinear degree <500), the accuracy cannot be improved. In order to obtain more accurate wave heights from the radar data, the degree of nonlinearity should be calculated from the radar data in order to determine a suitable method before estimating the wave height.

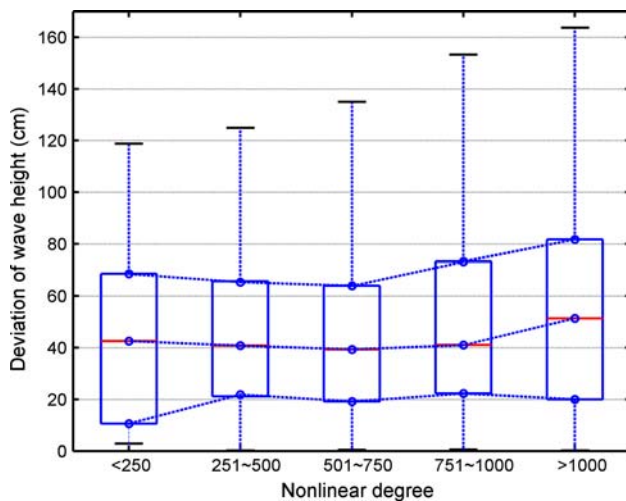


Fig. 9 The box-and-whisker plot for the degree of nonlinearity and the deviation between radar and data buoy observations. The signal-to-noise ratio method is applied for calculating the wave height from radar data. The maximal deviation of every box-and-whisker plot is also related to the degree of nonlinearity

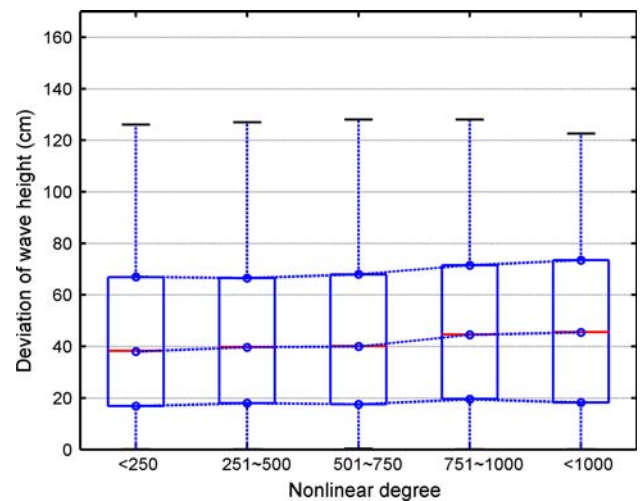


Fig. 10 The box-and-whisker plot for the degree of nonlinearity and the deviation between radar and data buoy observations. The modulation transfer function method is applied for calculating the wave height from radar data instead of the signal-to-noise ratio method

Conclusion

We have identified an obvious energy deviation from a linear dispersion relation curve in the spectra of X-band radar image sequences. The nonlinearity of radar backscatter is one of the factors that causes this deviation. In order to understand the influences of nonlinearity on radar observations of waves, the theory of bispectral analysis was applied to determine the degree of nonlinearity and features from radar backscatter series.

By analyzing the radar backscatter series from different water depth areas, we found that the harmonic energy peak is located in a higher spatial frequency domain in the case of shallow water areas compared to deeper water areas. Moreover, the degree of nonlinearity in shallow water is also higher than in deeper water. This result is similar to the bispectral results derived from analyzing water level time series.

The causes of nonlinearity are complex. The relationship between the degree of nonlinearity, meteorological and marine factors has been investigated. We found that the average degree of nonlinearity in radar backscatter is related to wind speed and wave height. The degree of nonlinearity derived from radar backscatter increases simultaneously with wind speed and wave height. The degrees of nonlinearity determined from the radar data are scattered at Beaufort wind scale 4 and scale 5 in wave height.

We investigated the relationship of the wave height accuracy determined by radar observation and the degree of nonlinearity calculated from radar backscatter. Our study concludes that the higher the degree of nonlinearity detected, the better the error detection observed by the radar. The method of modulation transfer function was applied for improving wave height accuracy by eliminating the influence of nonlinearity. The accuracy of estimating wave height from the radar can be improved only in the case of larger degrees of nonlinearity. We suggest calculating the degree of nonlinearity from radar data in order to determine a suitable method to accurately estimate wave height.

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