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A STUDY ON THE JOINT PROBABILITY OF WAVES AND WATER LEVELS DURING TYPHOONS

by

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ABSTRACT

Wave and water level are important factors to induce coastal flooding. Sea dyke is still the main construction to defense the sea impacts. The estimation of sea dyke height is therefore an important issue. They are commonly estimated by summation of significant wave height of certain return period, astronomical tide, storm surge. This is due to the assumption of max. significant wave heights and the water levels happen simultaneously. The objective of this paper is to study the probability and uncertainty of joint occurrences of high waves and high water levels. 111 typhoons data sets were collected from four field stations around Taiwan Waters. For the traditional frequency analysis method (FAM), we find the best model to simulate the significant wave height is Weibull Distribution. The best model for simulating water level is Extreme Type I Distribution. Traditional design wave heights and water levels are thus estimated by various frequency years. In this paper, the joint probability of wave height and water level was estimated by joint probability method (JPM). Comparative result shows that the total water level estimated by FA is lower than it estimated by JPM under the same occurrence probability. Monte Carlo simulation method was used to simultaneously simulate large amount data of significant wave height and water level, in order to assess the uncertainty of the joint probability of wave height and water level. The coefficient of variation (COV) is used as the uncertainty index. We find the uncertainty of joint probability did reduce by increasing the simulation data. However, the amount of field data is required to improve the simulation accuracy. The simulation result shows that when the field data amount is more than 1000, the uncertainty of joint probability of wave height and water level is less than 10%. This is a useful information for further study when one has to collect field data.

Keywords: Joint Probability Method, Frequency Analysis Method

1. INTRODUCTION

The traditional design approach to the coastal defense is essentially deterministic and empirical. The determination of the required dike height is based on the accumulation of maximum historically records from the high water levels due to spring tides, wind effects and the expected wave run-up or empirically takes the 100-year return period wave height and 10-year return period water level due to surge and tide as the 100-year return period joint event, for example. However failure of coastal protection due to flooding is mainly caused by a combination of high water levels and waves. If one considers these as independent variables, the probability of failure can be calculated easily, but this would be an incorrect assumption. These two variables are dependent, so the method suitable for multivariate statistics should be developed, if one does not want to over- or underestimate the probability of failure. Therefore when one is designing coastal defenses, such as sea walls, groynes, breakwaters etc., one should look at the likelihood of both conditions occurring simultaneously. A joint probability study of wave heights and water levels will do this and is an important research topic. Many studies (Ferreira and Soares, 2002; Battjes and Groenendijk, 2000; Memos and Tzanis, 2000; Prevosto et al., 2000 and Song et al., 2004) put focus on the joint distribution of wave height and period or sea surface elevations of two points in the sea. They provide more understanding on the scientific problem. However, the joint probability problem of wave height and water level is necessary on the design requirement of coastal defense engineering. Here in this study the wave height means the significant wave height (SWH). The water level (WL) is the summation of astronomical tides and meteorological residues however without the effect of waves or wave setup.

There are two approaches to construct the joint distribution of two variables. One is the analytical approach; the other one is the illustrative approach. If the respective distributions of wave height and water level are known as well as their dependent correlation is found, their joint distribution model can

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be found by statistical derivation. However, it is very difficult to find the determinate correlation between SWH and WL since their correlation may not be functioned. The best way to find their joint distribution is to use the illustrative approach. HR Wallingford (2000) presented the joint distribution of SWH and WL in UK. Rodríguez et al. (1999) presented the cases in Spain. The problem in the illustrative analysis of joint distribution is that the field data is always insufficient. Hawkes et al. (2000) used Monte Carlo to generate the data. Li and Song (2006) use a third-generation wave model and a 3D flow model to simulate long term data for analysis of the joint probability of wave height and storm tide level. This kind of probability-based design concept has been presented by Plate and Duckstein (1988) on hydraulic engineering long time ago. This concept is also used on coastal engineering (Liu et al., 2000) however it is still less.

In this paper, the study will be focused on the joint probability analysis of significant wave heights and water levels for typhoon data since the severe sea state induced by typhoons is the main impact for coastal defenses. By the traditional method on sea dike design, extreme wave height and water level are summarized to be the design value. This is on the assumption of the two events occurring simultaneously. This paper will study the joint probability of occurrence of these two variables, and its uncertainty. The joint distribution of significant wave height and water level for typhoon cases will be presented in illustrative form. Results of design example of sea dike will be compared by joint probability method (JPM) with traditional frequency analysis method (FAM).

2. TYPHOON DATA

Hourly time series of water levels and significant waves are used in this paper. They are from four in-situ stations, named Longdong, Hualien, Dapenwan and Eluanbi that locate at northern, eastern, southern and western of Taiwan respectively, in order to study the joint distribution form of SWH and WL by locations. Table 1 shows the basic data of the in-situ stations and the data amount used in this study. Locations of the sites are shown in figure 1. The wave data are measured by pitch-and-roll buoys. They locate at the water depth of 25 to 45m. The water level records are obtained from acoustic sensors in the wave-filter tube. There are only astronomical tide and storm surge measured. All data used in this study are quality checked.

Figure 2 is an example showing the hourly time series of significant wave height and water level at Hualien during typhoon Dujuan in 2003, as an example. From this case, the maximum significant wave height occurred when the water level was in the low. When there is the high water level period, the sea-state is not as severe as it at other time. This case shows clearly that the high water level is not always occurring together with high waves.

The data used in this study were collected during typhoons which moved forward to Taiwan and were alarmed to the public by Central Weather Bureau from year 2001 to 2005. The sea state is normally strong affected by a typhoon for one to three days because of the moving speed of typhoon is mainly from 10 km/hr to 25 km/hr, such as the example shown in figure 2. One data set composed of the maximum significant wave height and its corresponding water level in one typhoon are culled for analysis. Figure 3 shows the culled wave and water level data which will be analyzed in this study. Typhoons are seemed as independent weather systems. Therefore, data from these stations were pooled in order to extend the data amount. Totally, 111 typhoons producing 7440 simultaneous significant wave heights and wave levels are analyzed in this study.

Station	Data Category	Instruments	Water depth	Typhoon number	Data amount
Longdong	wave	Pitch-and roll buoy	32m	20	1353
	water level	acoustic sensor			
Hualien	wave	Pitch-and roll buoy	30m	32	2258
	water level	acoustic sensor			
Dapenwan	wave	Pitch-and roll buoy	25m	19	1241
	water level	acoustic sensor			
Eluanbi	wave	Pitch-and roll buoy	45m	40	2588
	water level	acoustic sensor			
			summation	111	7440

Table 1: Information of in-situ stations where the data analyzed in this study

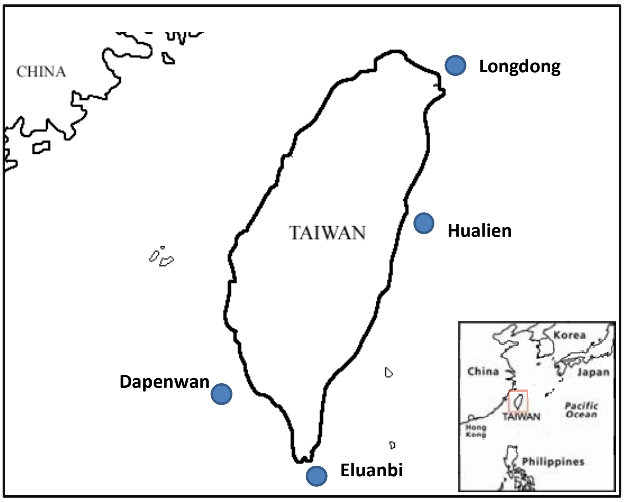

Figure 1: Locations of in-situ stations

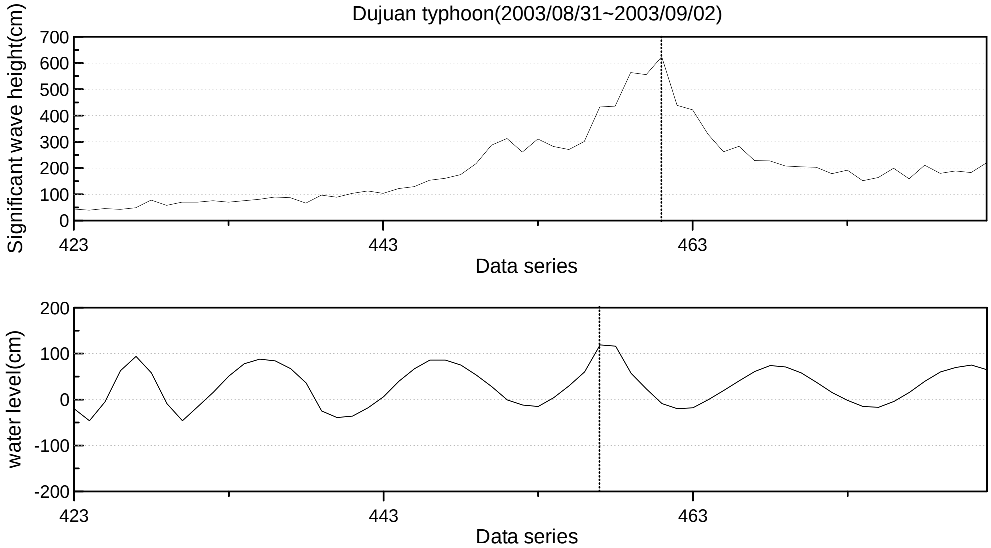

Figure 2: Example of simultaneous observation of significant wave height and water level during a typhoon (station: Hualien)

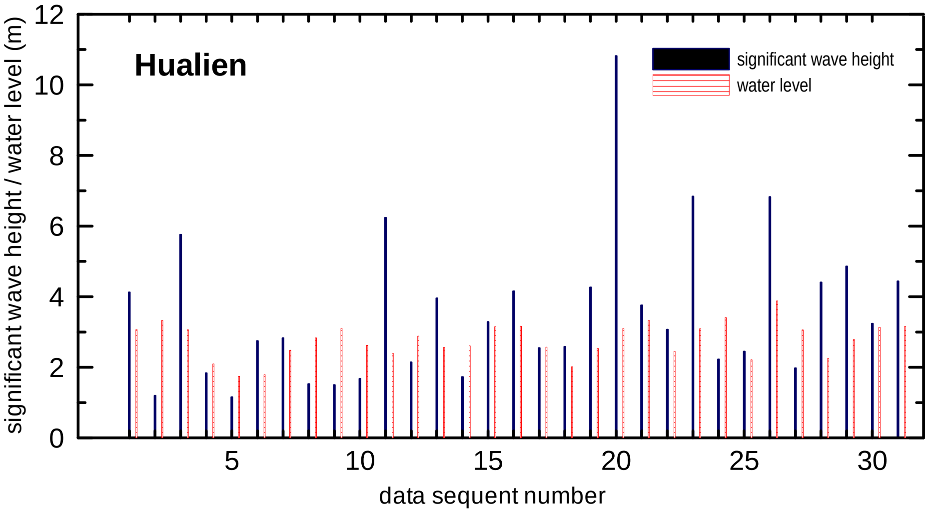

Figure 3: The maximum significant wave heights (left line) and their corresponding water level (right line) during all typhoons (Station: Hualien, 32 typhoons)

3. FREQUENCY ANALYSIS METHOD (FAM)

Hydrologic systems are sometimes impacted by extreme events, such as severe storms and floods. The magnitude of an extreme event is inversely related to its frequency of occurrence, very severe events occurring less frequently than more moderate events. The objective of frequency analysis of hydrologic data is to relate the magnitude of extreme events to their frequency of occurrence through the use of probability distributions. Since the duration of field observation is short, the data amount is always limited. Some approaches are developed for data generation. Hawkes and Hague (1994) use Monte Carlo simulation to generate large amount data for statistical analysis. Li and Song (2006) use wave and flow models to generate long-term data. Since this paper put focus on joint probability analysis of typhoon cases, the data is insufficient. The data analyzed for frequency analysis are assumed to be independent, so the maximum significant wave height and its corresponding water level of each typhoons are assumed as independent annual data. More than 30 typhoons data observed at four stations around Taiwan are used in this study. The objective of this section is to understand the design value of wave and water level on various return periods by traditional frequency analysis approach. The results are used for comparison with joint probability method presented in next section.

Marginal distribution models and statistical tests

A probability distribution is a function representing the probability of occurrence of a random variable. By fitting a distribution to the data, a great deal of the probabilistic information in the sample can be compactly summarized in the function. There are a lot of distributions presented in the text book (Ang and Tang, 1975; Hahn and Shapiro, 1967). In this study, following common used distributions are adopted. They are 3-parameters Log-Normal distribution (LN3), 1-parameter Rayleigh distribution (RL1), 3-parameters Weibull distribution (WB3), 3-parameters Gamma distribution (GM3), type I extreme value distribution (EV1) and the General Extreme Value Distribution (GEV). Two goodness-of-fit measures were introduced in this paper. They are Chi-square test (C-S test) and Kolmogorov-Smirnov tests (K-S test) (Hahn and Shapiro, 1967). In addition, the value of non-dimensional root mean square error (RMSE) of the distribution fitting was used to conclude the best model (Doong, 1996). These distributions were applied to simulate the synthetic data of significant wave heights and water levels during typhoons.

Table 2 shows the diagnosis of models fitting to the synthetic significant wave height and water level data. From the table, we know none of the distributions fit to the synthetic wave height within present two goodness-of fit. The 3-parameters Weibull Distribution (WB3) was therefore selected as the best one referring to the minimal RMSE. For wave level data, the best fit model is selected as Extreme Value Distribution Type I (EVI). Figure 4 shows the histogram of the synthetic significant wave height and the best fitting distribution. The histogram and best fitting model for synthetic water levels are shown in figure 5.

	significant wave height			water level		
	C-S test	K-S test	RMSE	C-S test	K-S test	RMSE
ND3	F	P	11.68%	F	P	4.17%
EV1	F	P	18.88%	P	P	3.23%
GEV	F	P	13.92%	F	P	3.89%
GA3	F	P	7.19%	P	P	4.69%
RL	F	P	13.54%	P	P	4.35%
WB3	F	P	7.07%	F	F	3.65%

Table 2: Diagnosis of models fitting to the wave height and water level data of synthetic data

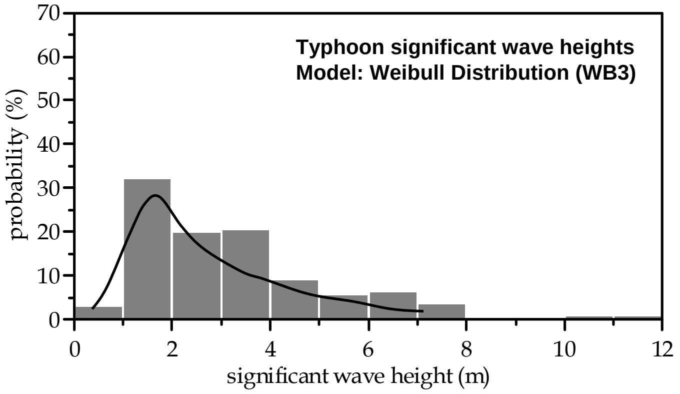

Figure 4: Distribution model fitting for synthetic significant wave height time series

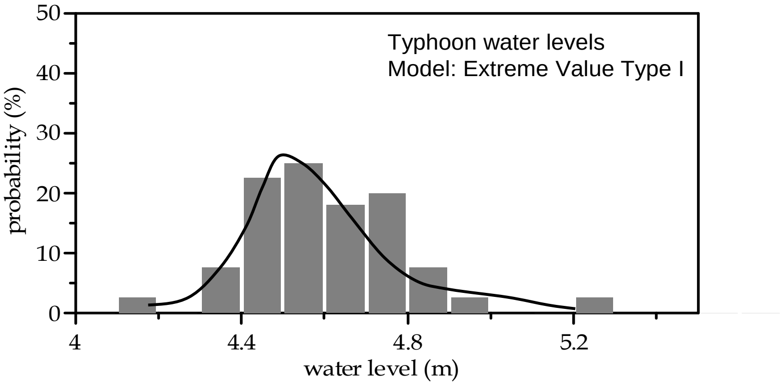

Figure 5: Distribution model fitting for synthetic water level time series

Design value of return periods

The objective of frequency analysis is to evaluate the design value of various return periods. The return period refers to the average period of time between occurrences of a particular high value of that variable. This design value is useful for design of dyke height on the purposes of flood protection at river or coast. The design values of significant wave height and water level in this study are listed in table 3.

Station	Content	Return Period (year)				
		5	10	50	100	200
Longdong	SWH	7.71	8.72	11.84	12.11	13.14
	WL	5.38	5.45	5.63	5.78	5.88
Hualien	SWH	7.82	8.91	12.05	14.23	16.52
	WL	5.31	5.42	5.56	5.72	5.81
Eluanbi	SWH	8.24	9.5	12.63	13.51	15.45
	WL	5.45	5.55	5.77	5.90	6.01
Dapenwan	SWH	6.31	7.24	9.15	10.42	11.43
	WL	5.26	5.35	5.45	5.55	5.61
Synthetic Data	SWH	6.85	8.32	10.54	11.81	13.15
	WL	5.36	5.43	5.60	5.79	5.84

Unit: meter

Table 3: Design values of significant wave heights (SWH) and water levels (WL) by FAM

4. JOINT PROBABILITY METHOD (JPM)

Methodology

When assessing the probability of failure for a single sea condition variable, the events which give failures are easily characterized as failures of the structure caused by extreme value of the variable, i.e. failures occur whenever the variable exceeds some level. The frequency analysis presented in last section is one of the approaches. When the sea condition variable is inherently multivariate, the joint probability approach is applied. JPM is an approach for estimating the probability of a structure variable exceeding a critical level, based on the joint analysis of the sea condition variables. The joint probability typically refers to two or more partially related environmental variables occurring simultaneously to produce a response of interest, such as large wave heights and high water levels, large river flows and high sea levels, large surges and high astronomical tidal levels.

The failure probability is critical in the evaluation of the reliability of a structure. Traditional, the failure probability of a coastal structure to waves or water level (storm surge) can be obtained from a function (distribution) by a given value of the function (failure condition). For assessing the failure probability of multivariate environment factors, the failure probability (exceedence probability of occurrence) is determined by integrating the joint probability density function over the failure region. As illustrated in figure 6, with the same return period, the joint exceedence probability may be smaller than the failure probability because the structure may fail when one of the parameter (wave height or water level) is high but the other is low.

Structure fails when the wave height and water level falls within the ‘failure region’ with boundary $b(x,\nu)=0$, where ν denotes the structural parameters and $x=(x_1,x_2)$. $b(x,\nu)=0$ indicates the limiting state exceeding which the structure will be incapable to resist the environmental loads or flooding occurs. The boundary function is different for different structure and for different location. The failure probability is given by $F=\iint f(x_1,x_2)dx_1dx_2$ in which $f(x_1,x_2)$ is the joint probability density function and the return period $T=1/F$.

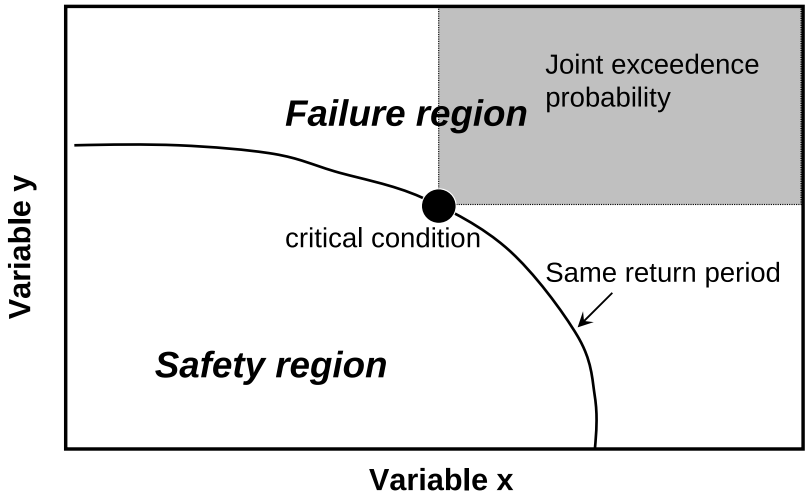

Figure 6: Illustration of the failure domain and the joint exceedance probability plotted on the graph of probability density function (HR Wallingford and Lancaster University, 2000)

Results

The results of joint probability distribution analysis are shown in figure 7. The scatter diagram in figure 7 is from the wave height and water level data. The curves of return period 5, 10, 50, 100 and 200 years are plotted in the figure. The curve means that the joint exceedance probability is 0.2, 0.01, 0.02, 0.01 and 0.005 when the sea state is more severe than it pointed at the ‘critical condition’. If one uses the critical condition as the ‘design value’, there are infinite solutions on the curve. For example, in figure 7(a) the exceedance probability is 0.01 (T=100) when the water level and significant wave height are respectively (5.5, 1.8), (5.0, 5.0), (4.5, 5.6)…and so on, however there is one outcome of 100 year wave height (12.11m, listed in table 3) and one outcome of 100 year water level (5.78m, listed in table 3). They are independent estimated. The height summation is therefore 17.89m on the consideration of water level plus wave impact. Figure 8 is the joint probability diagram for synthetic wave height and water level data. By joint probability analysis, we know there are infinite combinations of wave height and water level, i.e. there are infinite design heights. The combination is shown as the curve in table 4. It is shown that the summation of wave height and water level estimated by traditional frequency analysis is lower than it estimated from present joint probability method.

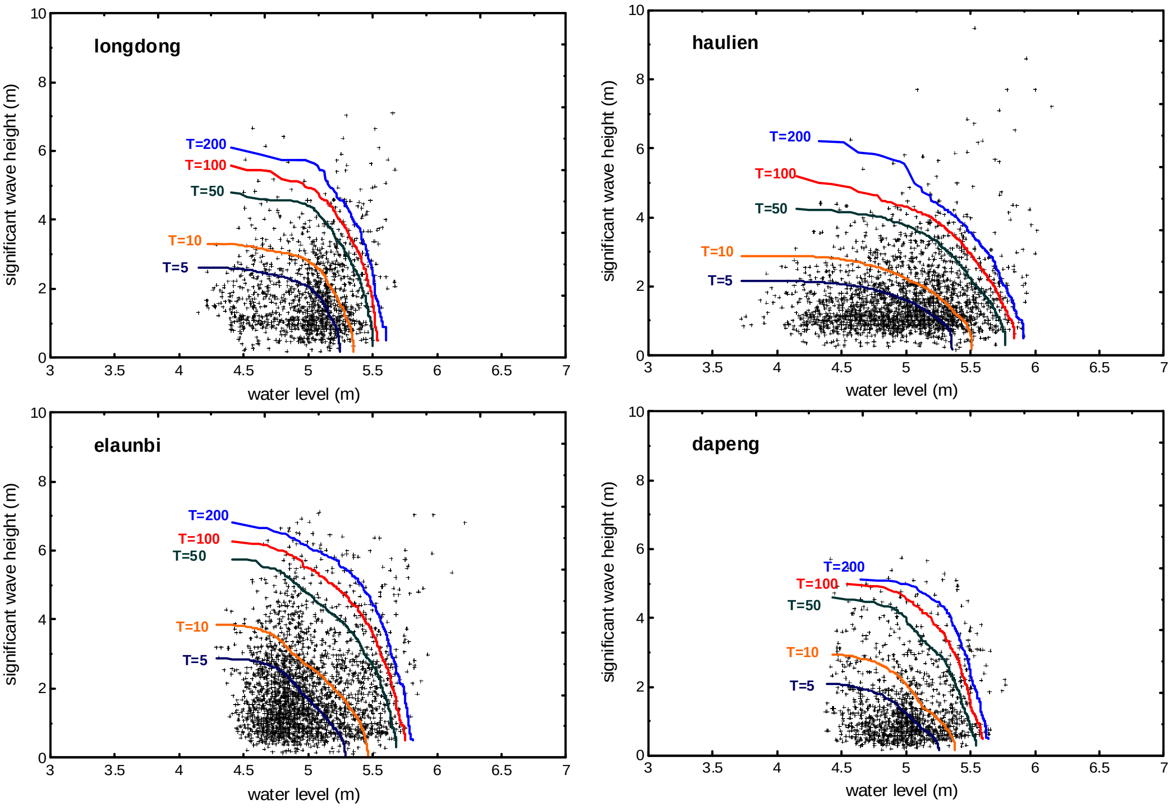

Figure 7: Diagrams of joint probability of wave height and water level (a)Longdong (b)Hualien (c)Eluanbi (d)Dapenwan

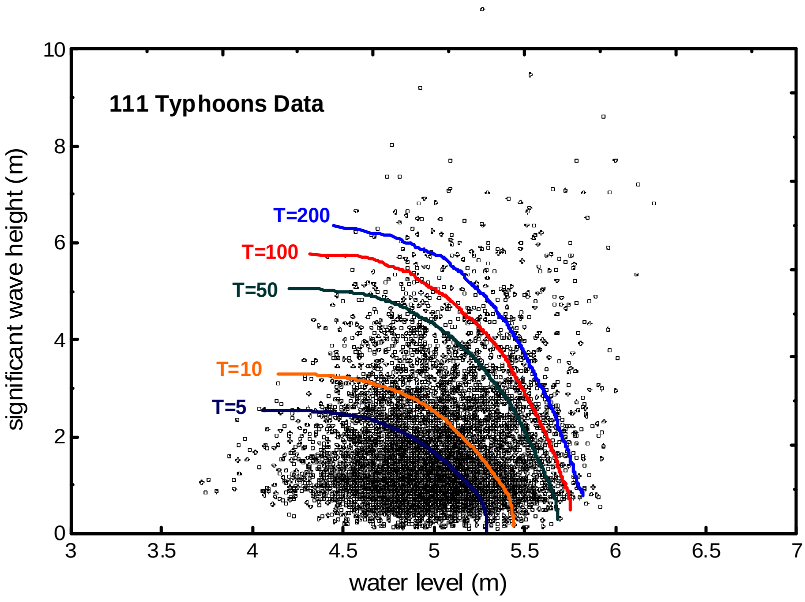

Figure 8: The joint probability diagram of synthetic wave height and water level data

	probability	Equivalent return period (year)	Design water level		
			FAM	JPM	
				min	max
$HS_{10}+WL_{10}$	0.01	100	13.75	6.49	13.20
$HS_{10}+WL_{50}$	0.002	500	13.92	6.85	15.73
$HS_{10}+WL_{100}$	0.001	1000	14.11	6.94	16.60
$HS_{50}+WL_{10}$	0.002	500	15.97	6.85	15.73
$HS_{50}+WL_{50}$	0.0004	2500	16.14	7.05	18.52
$HS_{50}+WL_{100}$	0.0002	5000	16.33	7.14	19.11
$HS_{100}+WL_{10}$	0.001	1000	17.24	6.94	16.60
$HS_{100}+WL_{50}$	0.0002	5000	17.41	7.14	19.11
$HS_{100}+WL_{100}$	0.0001	10000	17.60	7.16	19.63

Table 4: Comparison of summation of wave height and water level by traditional FAM (Frequency Analysis Method) and JPM (Joint Probability Method).

5. CONCLUSION

The greatest risk to coastal defense structures tends to occur at times of unusually high water levels combined with large waves. The overall effect of high wave conditions on coastal processes is highly dependent on the water level. For example, a severe typhoon coinciding with neap tide conditions would have less impact on beaches than a more moderate event in conjunction with a large spring tide. It has been shown that the waves and water levels should be jointly considered. Reliable estimates of the probability of occurrence of such combined conditions are given in this paper by joint probability method (JPM).

Distribution models are fitted to the data to find the best fitting function by statistical tests. The design value of specific return period (year) is evaluated. This traditional frequency analysis method (FAM) is an empirical model. Choice of a suitable distribution model is a critical factor on the calculation. This paper presents the illustrative approach to find the joint probability of two variables, such as significant wave height and water level in this study. From the analysis of joint probability method, infinite solutions are found for a specific failure condition. They are dependent on the conditions of wave height and water level. This is the advantage of the JPM method. One can consider which one is the important factor to decide the best solution. Since it is not suggested to compare the results of height summation of wave height and water level (design value) from FAM and JPM due to their different considerations, the results show that the height summation of wave height and water level by JPM is sometimes higher than it estimated by FAM but mostly locates between the results from these two methods. This is because the traditional frequency analysis uses the independent assumption on wave height and water level however they are dependent. This paper presents a new idea on the estimation of design height of sea dike. It is primarily proofed that the joint probability analysis is worthy to be have further studies.

Acknowledgements

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