# Spatial-Frequency Analysis of Drifter Data in the Western Pacific

*Hwa CHIEN[1], Jia-Lin CHEN[2], Pei-Hung CHEN[1], Chia-Chuen KAO[2]*

*1. Coastal Ocean Monitoring Center, National Cheng Kung University, Tainan, Taiwan*

*2. Dept. Hydraulic & Ocean Eng., National Cheng Kung Universitym Tainanm Taiwan*

## ABSTRACT

Lagrangian observation by drifters is one of the important approaches to realize the ocean surface currents. The averaged velocity of drifters could be obtained by the change of the GPS locations divided by time difference. Any series of current velocity can be further resolved into two time series, representing the *u* and v current components. Rotary spectral analysis (Gonella 1972) resolves, at each spectral frequency, the combined *u* and v components into two circular motions with opposite directions of rotation. Rrotary spectral analysis is a frequent applied method for recognizing currents. In the processes of rotary spectral analysis, the time-series data were first analyzed by fast Fourier transform (FFT), which gives a reversible transformation between temporal domain and the frequency domain. The drawback of FFT is that if the time series has duration of *T*, then the series is analyzed in terms of harmonics of the frequency 1/T, and further, little account is taken of time variation in the spectrum. In another word, the results of FFT analysis are time-mean representative, by which the analysis of inhomogeneous or non-stationary time series is constrained.

Tradictionally, surface drifters data are not usually used for tidal current analysis in general due to the fact that the latter typically requires relatively long time series at a fixed location in order to resolve individual constituents that have nearly the same frequencies. Moreover, In the coastal region or continental shelf with rugged bathymetry, the shallow water effect leads the ocean currents feature spatially inhomogeneous. The spatial distributions of the amplitude of the generated shallow water constituents vary with locations. Drifters usually pass through the coastal region less than a week and seldom stay more than a few days in a region where the tidal currents are uniform. Therefore, the main requirement of statistical analysis, stationary, is not upheld by drifter data. Difficulties of rotary spectral analysis in analyzing this kind of data arose as it is not capable of deeding with a time-invariant time series.

It is the aim of present study to retrieve the information of periodical oscillations of ocean currents from drifter's path data. We introduced The Wavelet Transformation to obtain Spatial-frequency characteristics of ocean currents. In this approach, the frequency, amplitude and corresponding phase of the periodical ocean current oscillation at any specific location in the drifter trial could be identified.

Case study is also performed in present study. The Drifter buoy launched from the south part of Taiwan at Aug. 2000 and was retrieved in Okinawa in Feb. 2001. GPS data of every 2 hours were recorded for analysis. The Wavelet Transform is utilized to derive the short time rotary spectral analysis that permit ocean surface drifter data to be used in the investigation of spatial variability of periodic currents and ocean circulations. The amplitudes of Constituents

of $O_1$、$K_1$、$N_2$、$M_2$ and $S_2$ are demonstrated. Besides the diurnal and semi-diurnal tidal currents, the drifter was taken by other current with longer periods. By comparing the traced of the drifter to the sea elevation anomaly data taken by satellite altimeter, it is found that the currents in the western Pacific are dominated by these Meso-scale eddies. Statistics of the current velocity are carried out. The spectral structures of the Meso-scale Eddy in the Western Pacific are also demonstrated. A 7.5 days of period, which is rather stable, did not vary with time and space, could be seen.

With the more condensed drifter observations over the ocean surface, it is the hope that the in-situ amplitude and phase information of the constituents could be obtained for any specific location in the ocean.

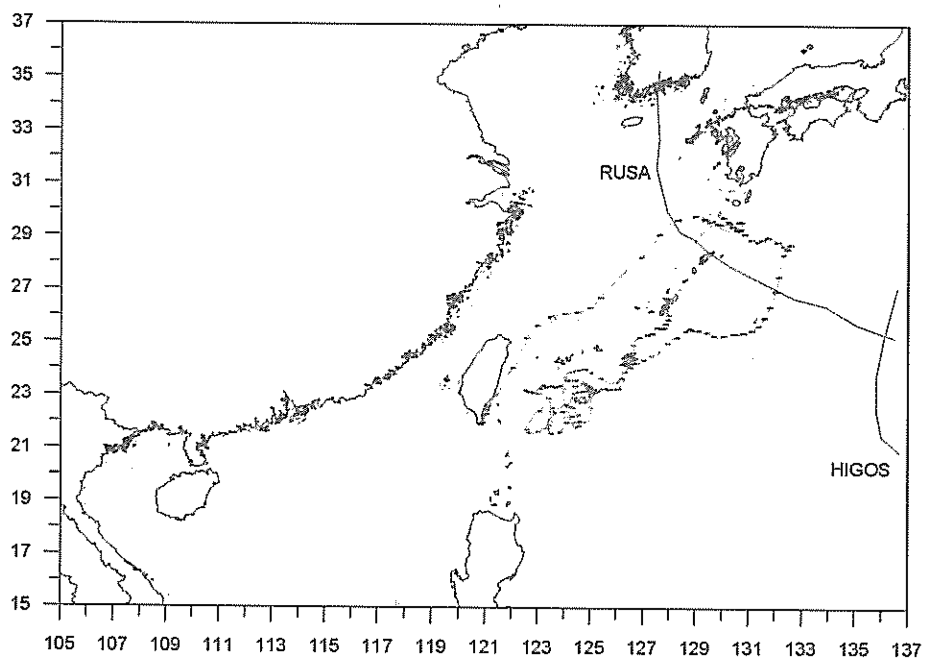


Fig.1 The path of the drifter.

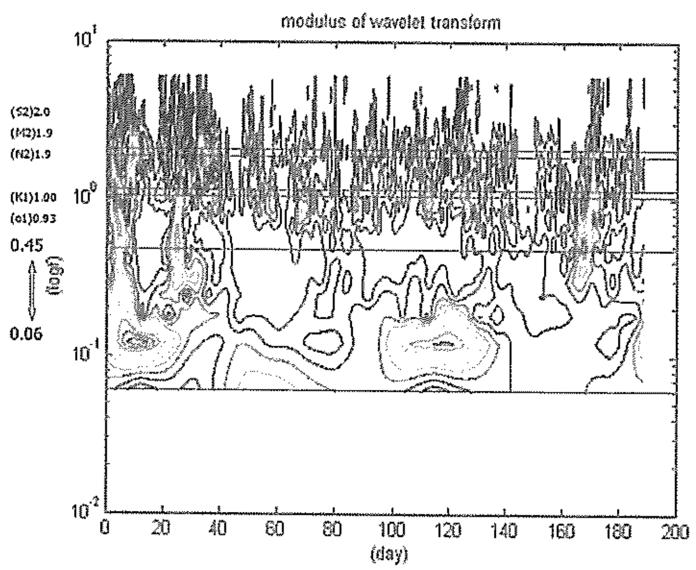


Fig.2 The temporal-frequency structure of ocean surface current in the western Pacific. In which the effects of semi-diurnal, diurnal constituents and low frequency Meso-scale eddies could be identified.



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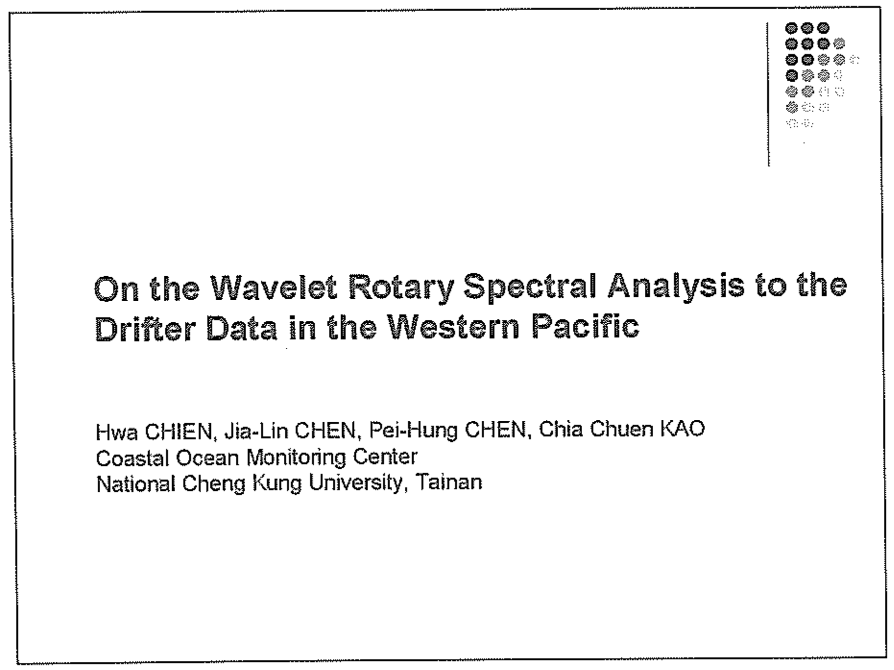


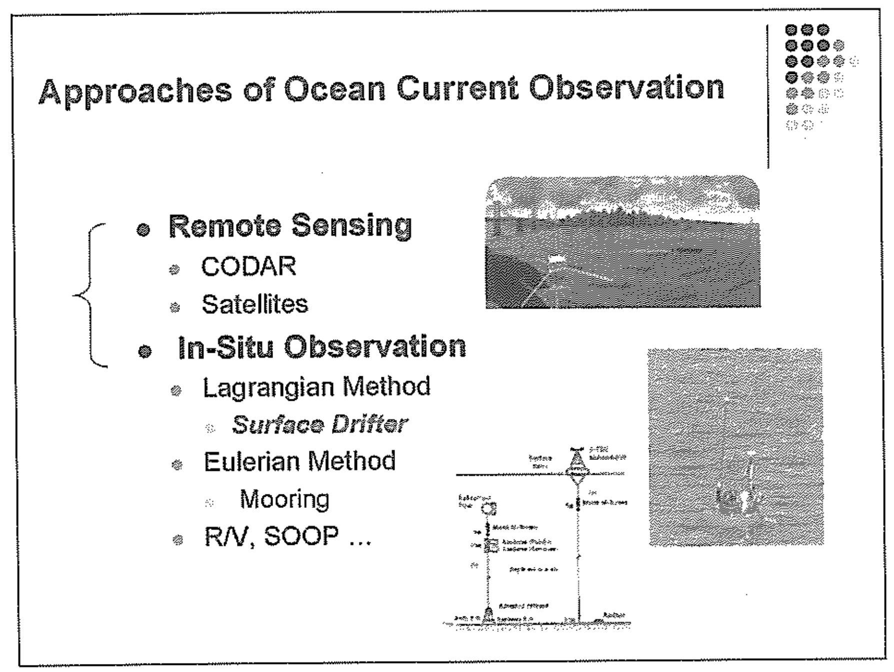


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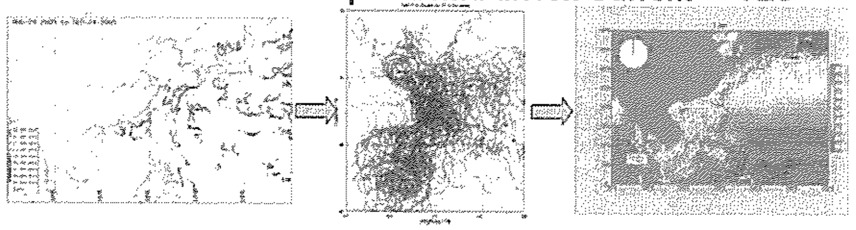
## Applications of Surface Drifter Data

•**Realization of Geostrophic\Wind driven Current** – YES



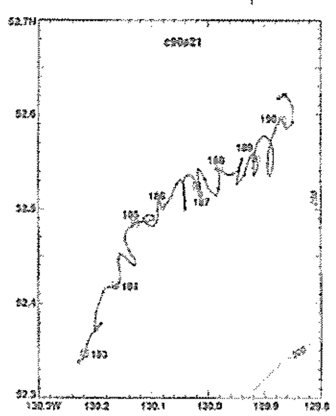
•**Realization of tidal current** – NOT YET

- Records at fixed locations are not long enough for resolving individual constituents
- Being affected by topography, tidal currents are uniform

⇨ ***The Drifter Data is*** Non-Stationary

## The Purpose of Present Study

- Purpose
  - Retrieve the information of amplitudes, phases of tidal currents and other periodic currents from drifter data
- Strategy
  - Develop techniques that is capable of dealing data with temporal and spatial variability







## Method of Drifter Data Analysis

- Random Vectors Analysis
  - Statistics: Bayesian, Principal Component Analysis ...
  - Spectral: Rotary spectral analysis
- Rotary Spectral Analysis
  - The most important studies Rotary spectral analysis are
    - Fofonoff (1969)
    - Gonella (1972)
    - Mooers (1973)
    - Calman (1978a,b)
    - Middleton (1982), and others

## Fundamental of Rotary Spectral Analysis

•Rotary Vector

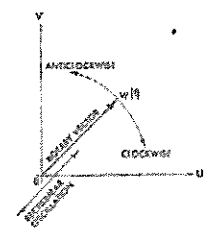
$w(t) = u(t) + iv(t)$

•Anti-clockwise rotary component

$w^+ = \frac{u - iv}{2}$

•Clockwise rotary component

$w^- = \frac{u + iv}{2}$



$$W_k^{\pm} = FFT(w^{\pm}) \Rightarrow U_k^{\pm} + iV_k^{\pm}$$

$$\begin{cases} U_k^{\pm} = \frac{1}{2}(CX \mp iSX) \\ V_k^{\pm} = \frac{1}{2}(CY \mp iSY) \end{cases}$$

Rotary velocity spectral amplitudes, *a*, and phases could be calculated as

$$a = 1/2[(CX - SY)^2 + (CY + SX)^2]^{1/2},$$
$$a = 1/2[(CX + SY)^2 + (CY - SX)^2]^{1/2},$$
$$\varepsilon = \arctan[(CY + SX)/(CX - SY)],$$
$$\varepsilon = \arctan[(CY - SX)/(CX + SY)],$$
$$\theta = \frac{1}{2}(\varepsilon + \varepsilon)$$

## Limitation of Fourier Transform

The integration can not be carried out until the **entire signal** is known

$$W(f) = \int_{-\infty}^{\infty} w(t) \cdot e^{-i2\pi f t} dt$$

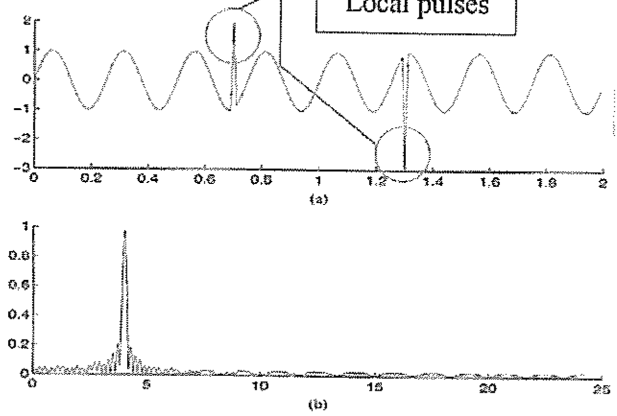
This is because the $e^{-i2\pi f t}$ or $\sin\omega t$ and $\cos\omega t$ are *global functions*

- The spectral structure obtained by FFT is a temporal (spatial)-mean representation

⇨ Fourier Analysis is invalid if stationary of data is not upheld

## An Example

Local pulses



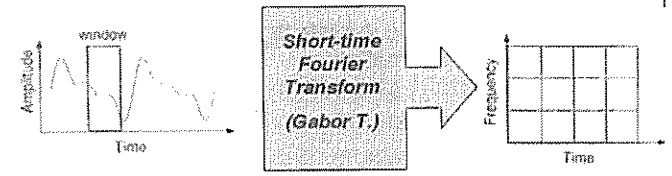

- No localization could be identified in Fourier Spectrum
- The effects of Local pulse is average Fourier Spectrum







## Short-Time Fourier Transformation (STFT)




⇨ $G_\phi f(b,\xi) := \int_{-\infty}^{\infty} f(t)\overline{\phi_{b,\xi}(t)}\,dt$

$\phi_{b,\xi}(t) := \phi(t-b)e^{j\xi t}$

- Non-adaptive window function
  - Limited spectral resolution for low frequency signal
  - Limited temporal resolution for high frequency signal

## Wavelet Transform

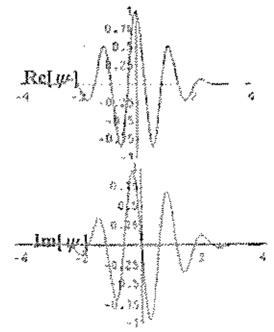
- $CWT(a,b) = \rangle \eta, \psi_{a,b} \langle = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} \eta(t) \cdot \psi^*\left(\frac{t-b}{a}\right) dt$

  - a: function of scale
  - b: function of position

$\hat{\psi}_{\text{Morlet}}(\omega) = \pi^{-1/4}\left[e^{-(\omega-\omega_0)^2/2} - e^{-\omega^2/2}e^{-\omega_0^2/2}\right]$

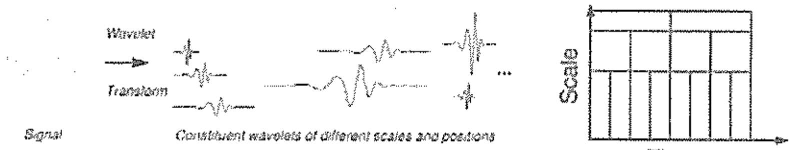
$\psi_{\text{Morlet}}(x) = \pi^{-1/4}\left(e^{-i\omega_0 x} - e^{-\omega_0^2/2}\right)e^{-x^2/2}$

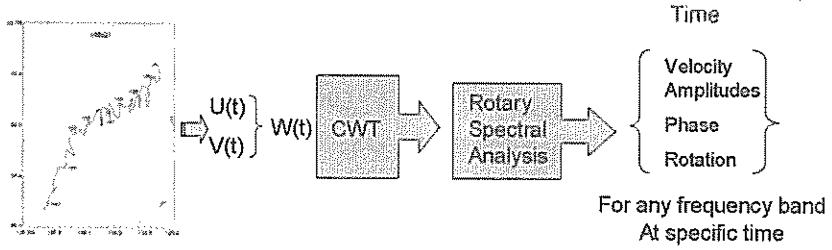
⇨ $\hat{\psi}_{\text{Morlet}}(0) = 0$



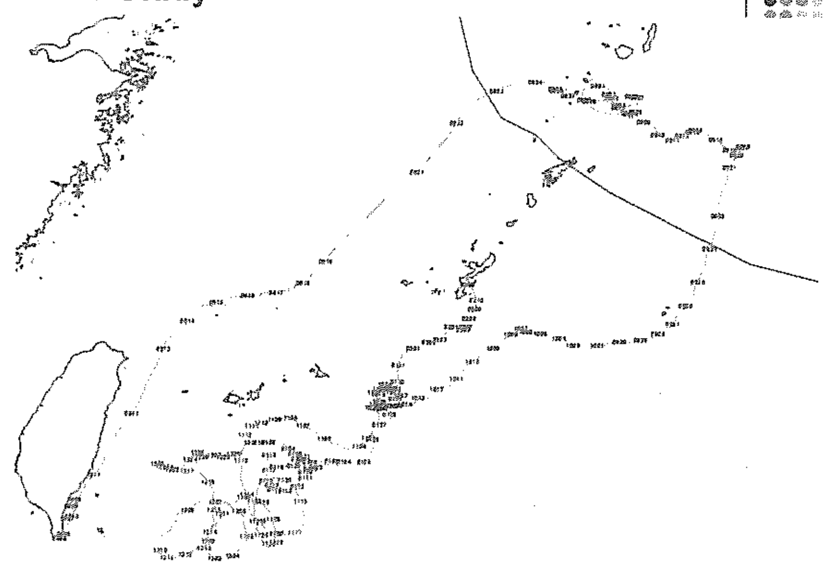
## The Temporal-Spectral Structure

- By giving the parameter a and b, the oscillation of mother wavelet could be changed in scale or translated to obtain the frequency and time information




## Case Study





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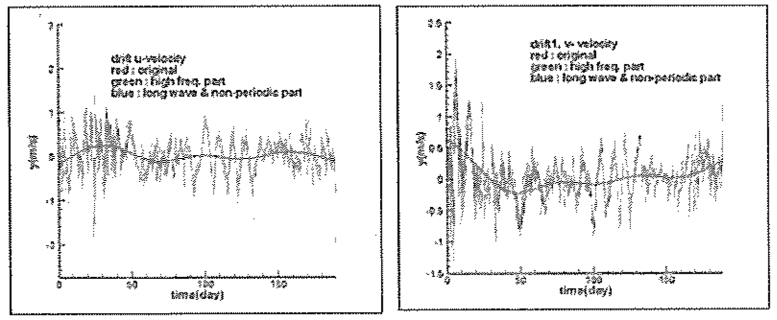
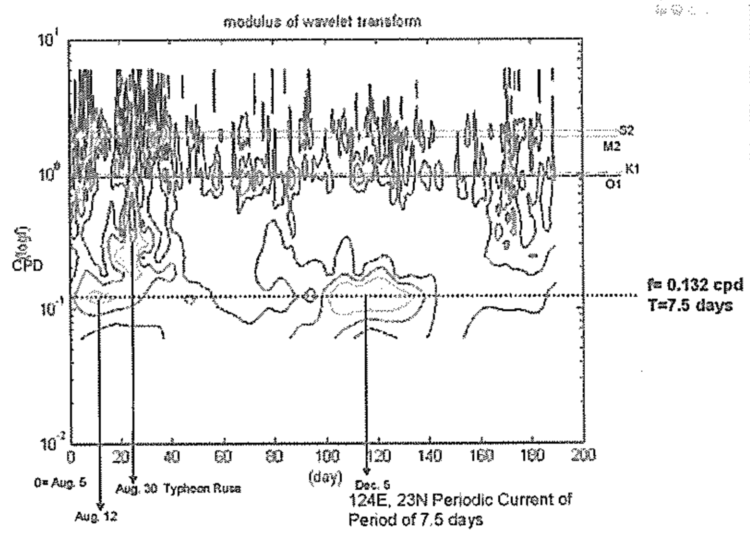
## Original Records of Velocity Components

- u,v of surface current speed




## P(W+) Anti-clockwise Component Wavelet Spectrum

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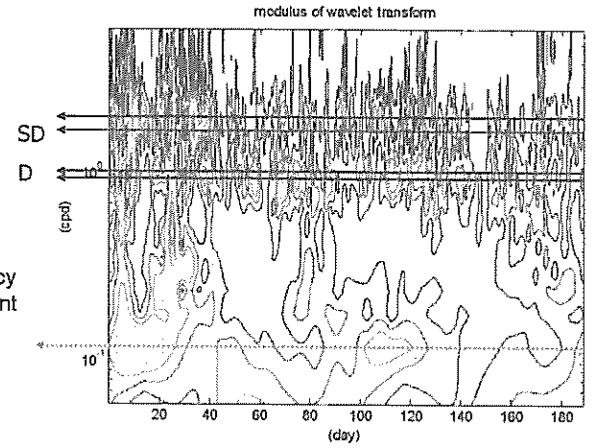
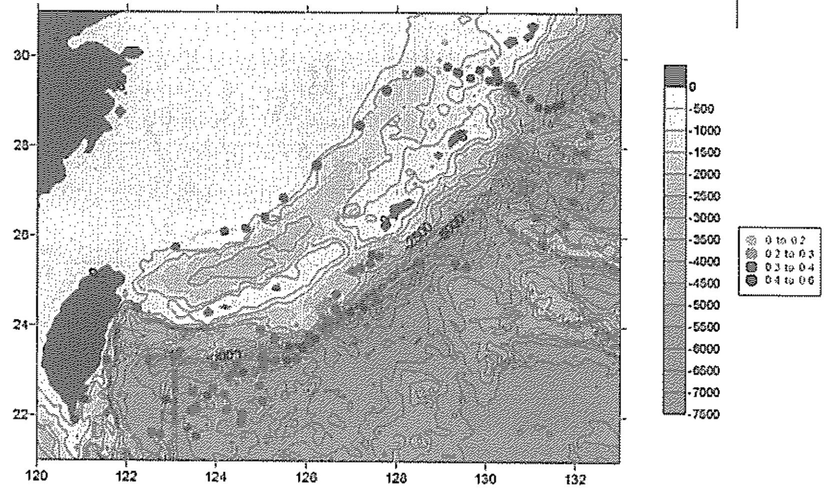
## P(W-) Clockwise Component Wavelet Spectrum




- Low frequency periodic current oscillation
- F=0.132cpd
- T=7.5 days

## The Amplitude of Diurnal Constituents along the Drifting Trajectory





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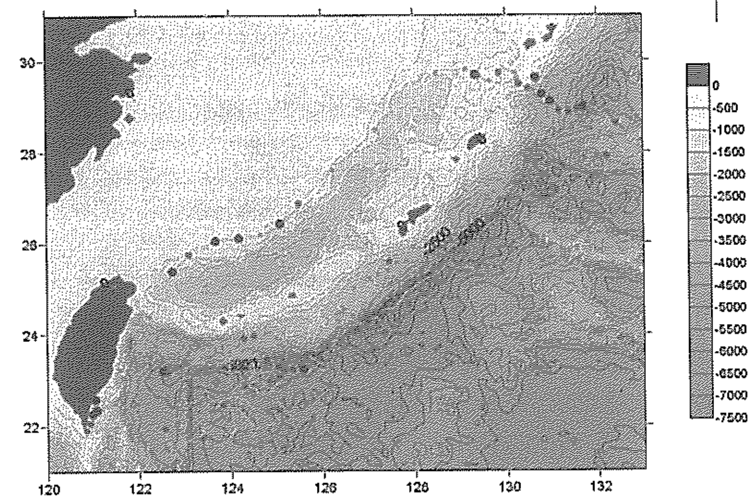
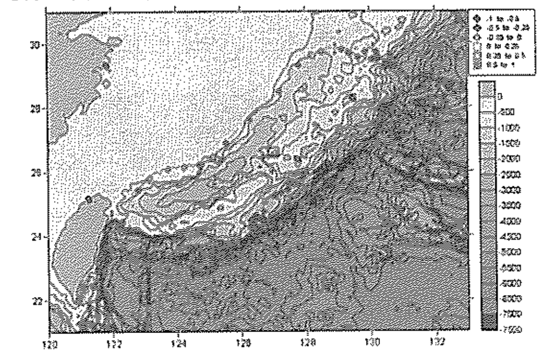
## The Amplitude of Semi-Diurnal Constituents along the Drifting Trajectory



## Rotary Coefficient along the Drifting Trajectory

- Rotary Coefficient could be calculated for any frequency at any time
  $CR(f) = (P_-(f) - P_+(f)) / (P_-(f) + P_+(f))$

  0<CR<+1 : Anticlockwise rotation
  CR=0 : Purely rectilinear motion
  0<CR <-1 : Clockwise rotation



The CR of the frequency band of 0.132 cpd

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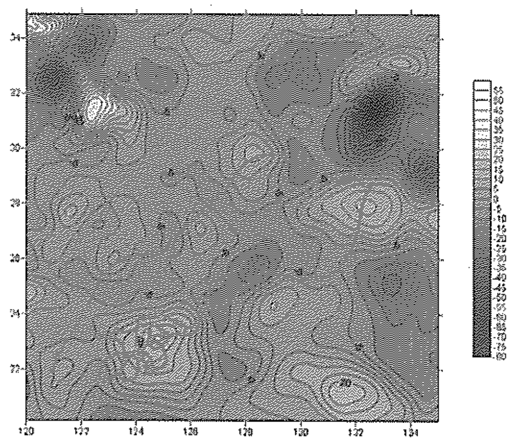
## The Low Frequency Current Oscillation Induced by Mesoscale Eddies

- Satellite Altimeter TOPEX/POSEIDON (T/P) measurements of Sea Surface Height Anomaly (SSHA) is used to compare with the drifting trajectory



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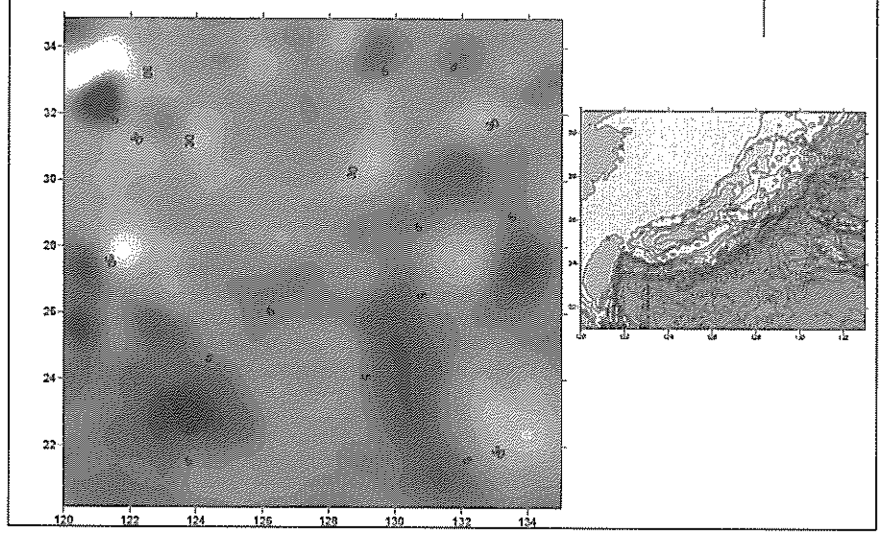







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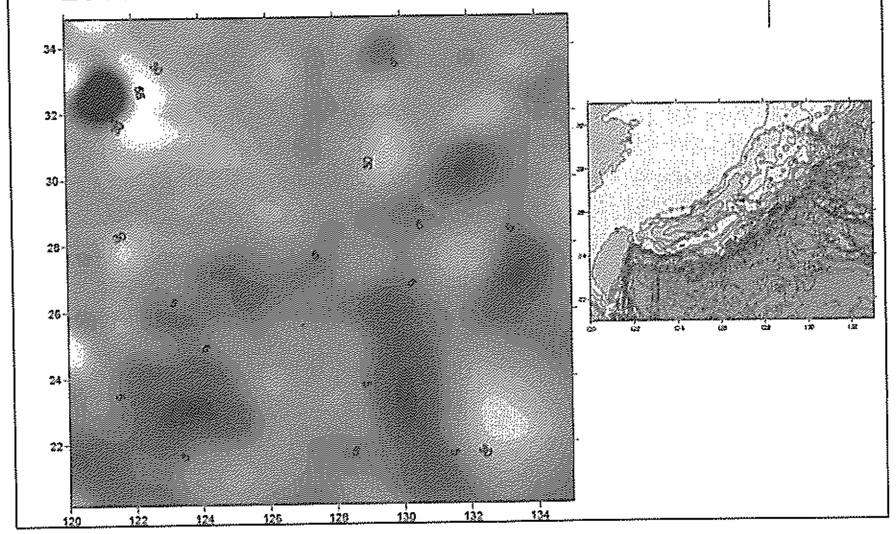

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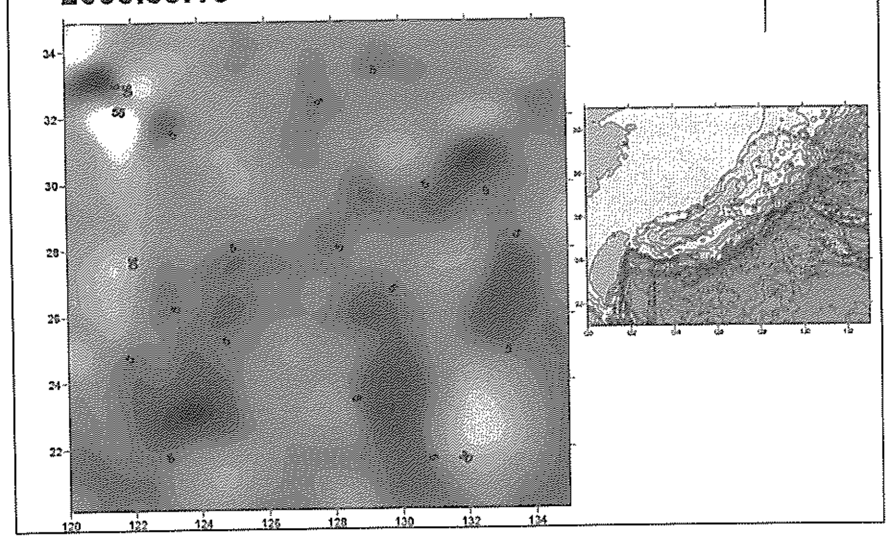

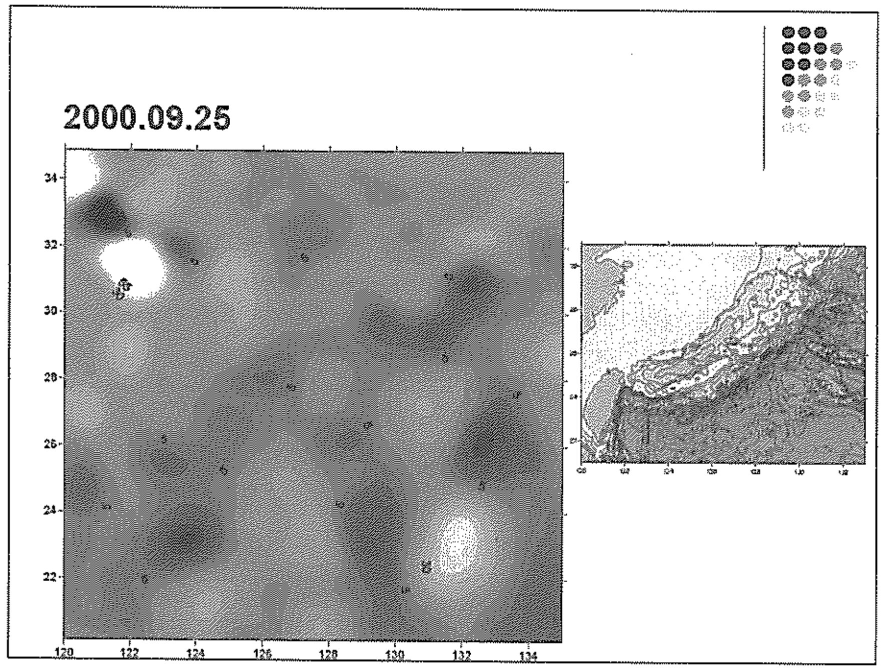
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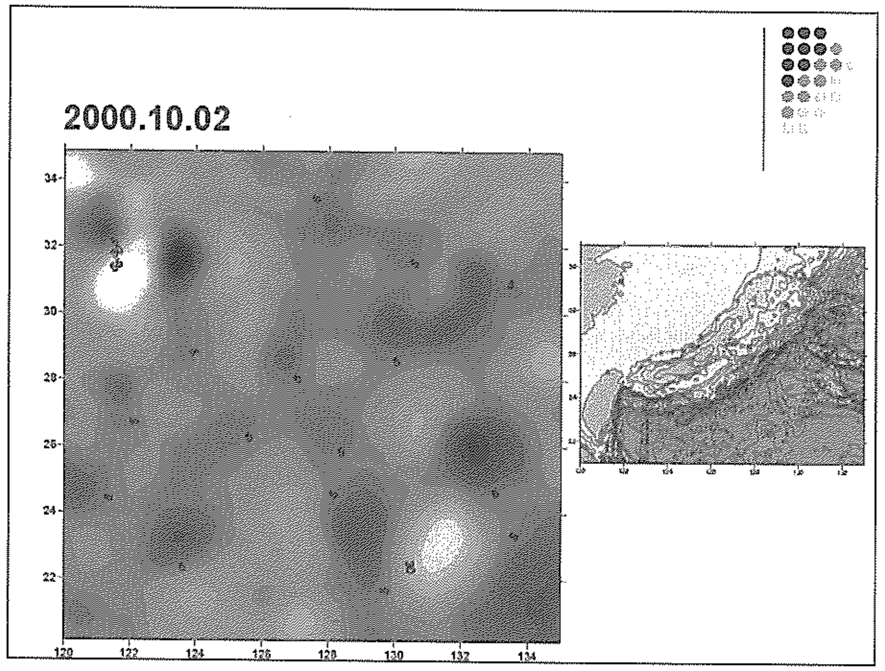
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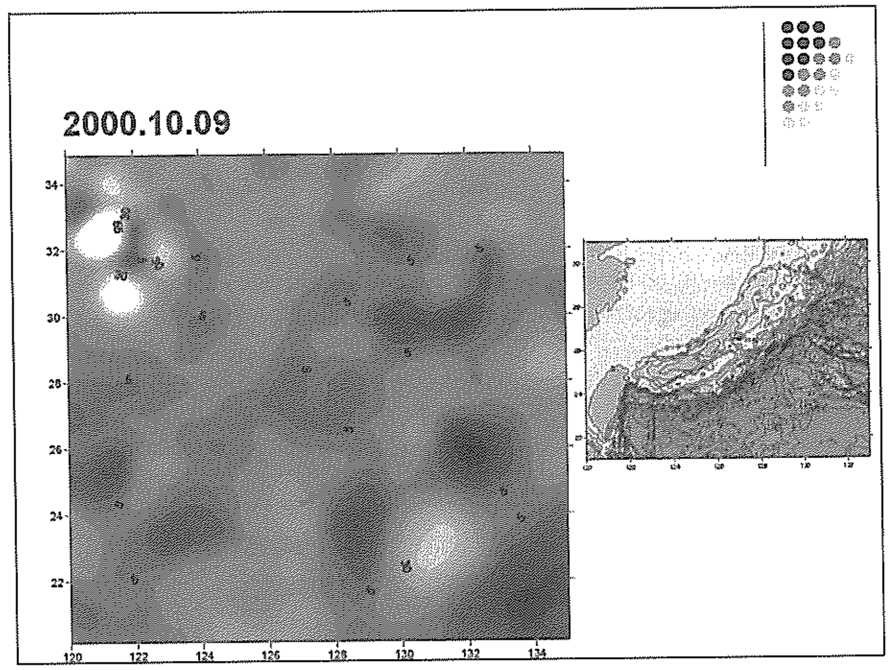
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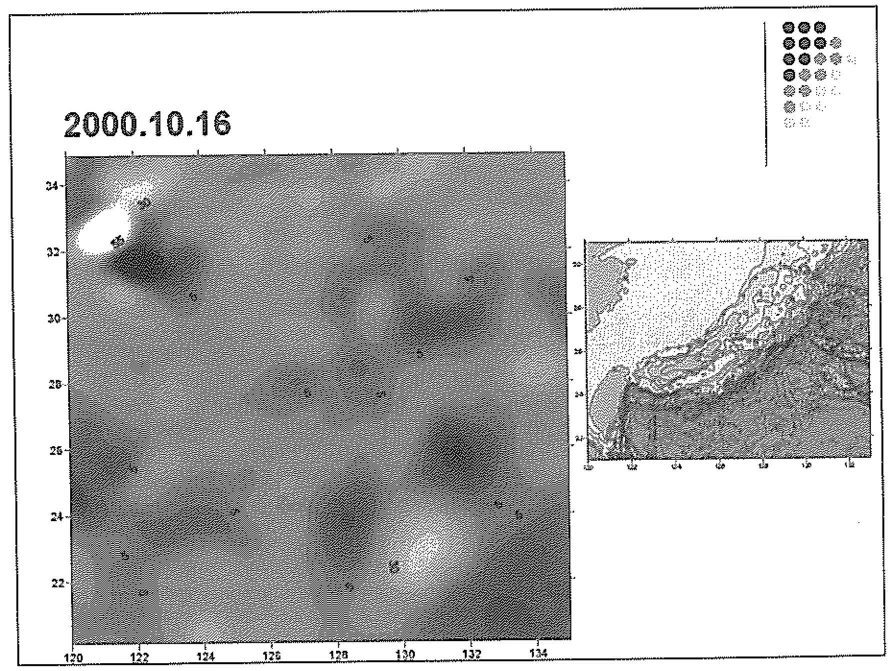
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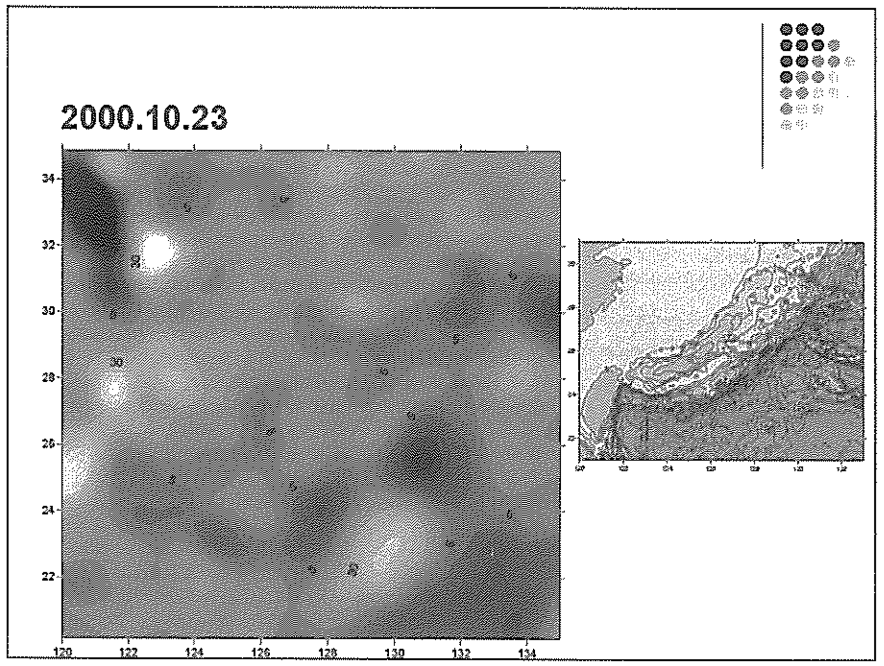
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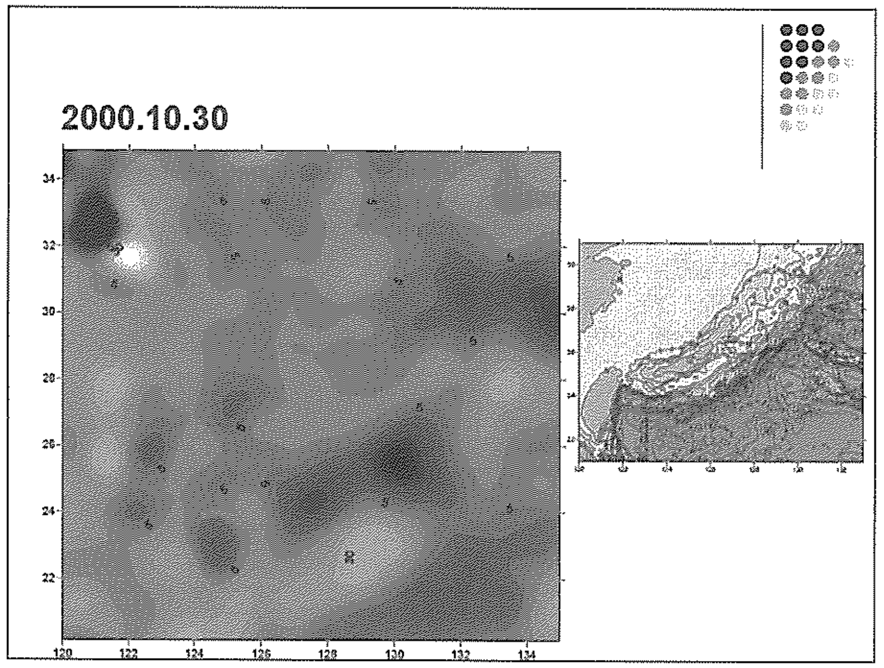
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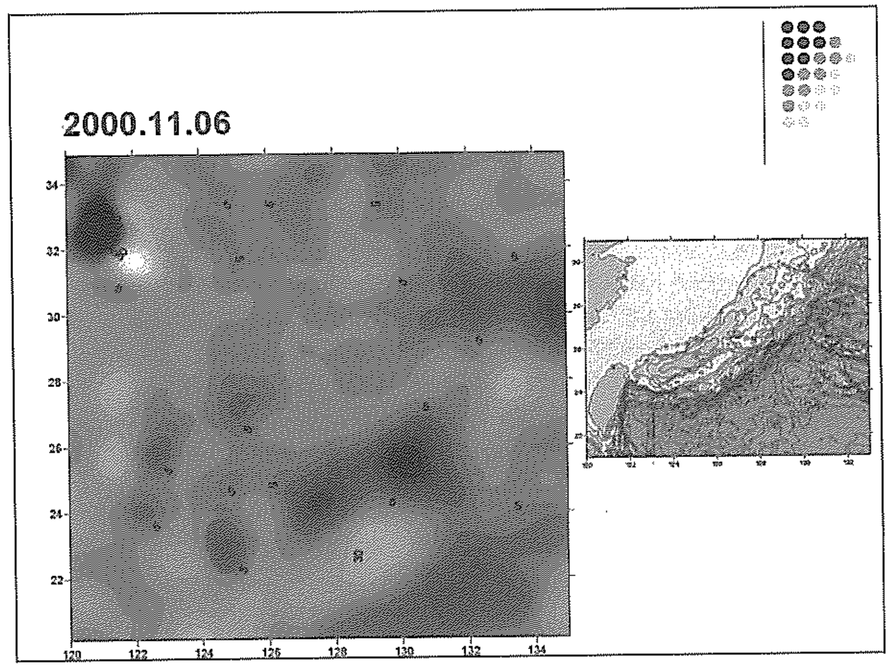
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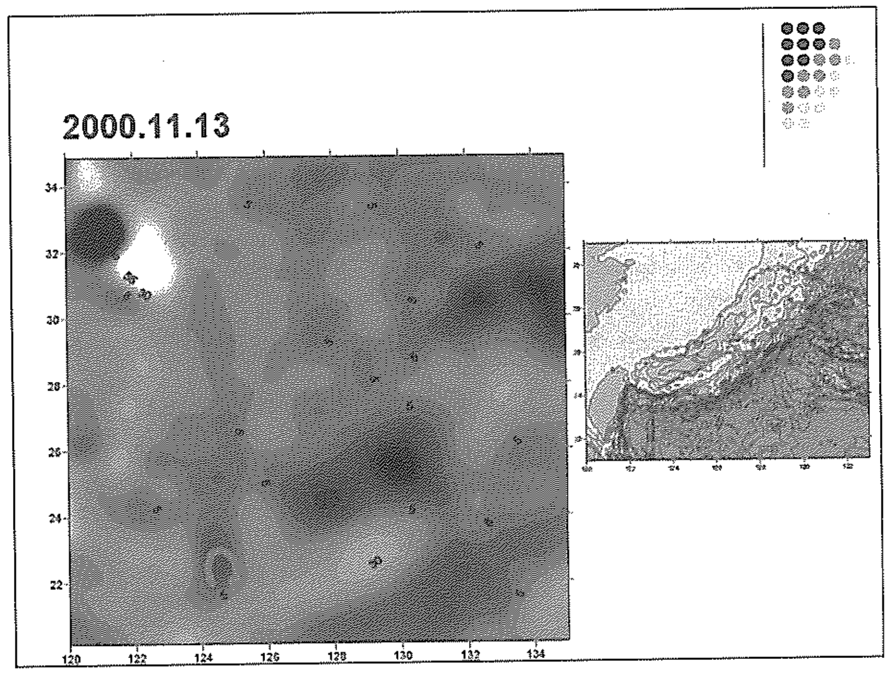
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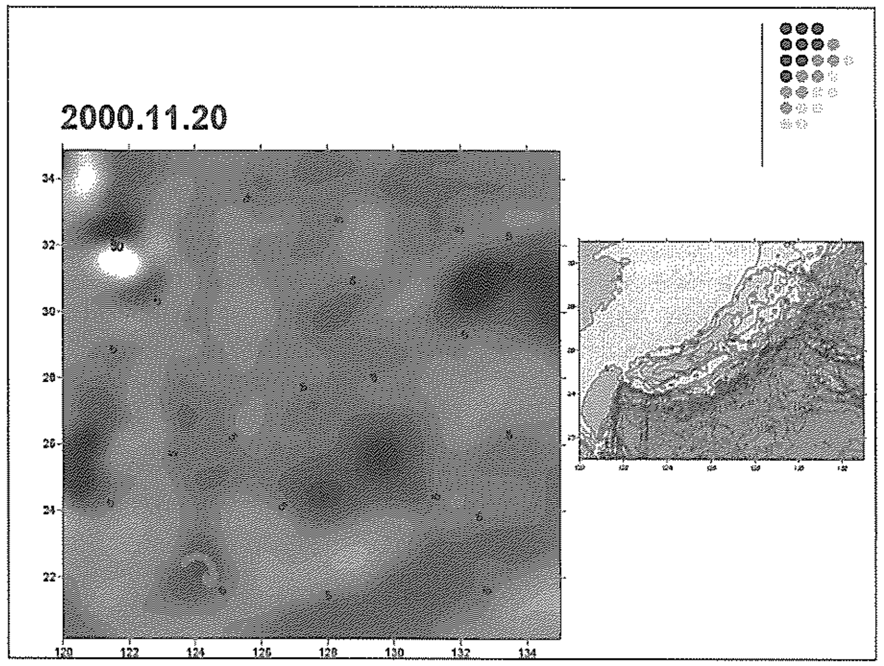
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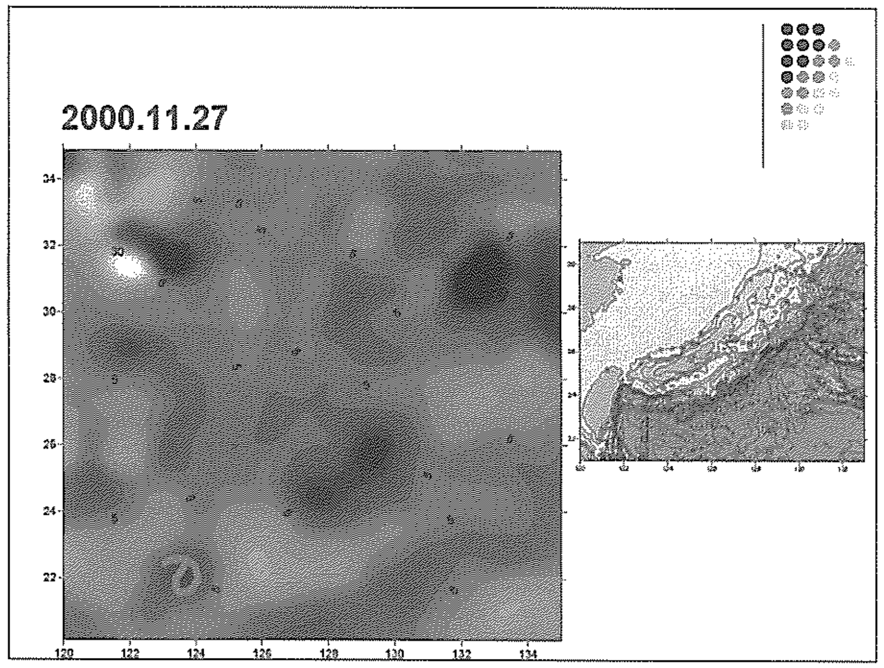
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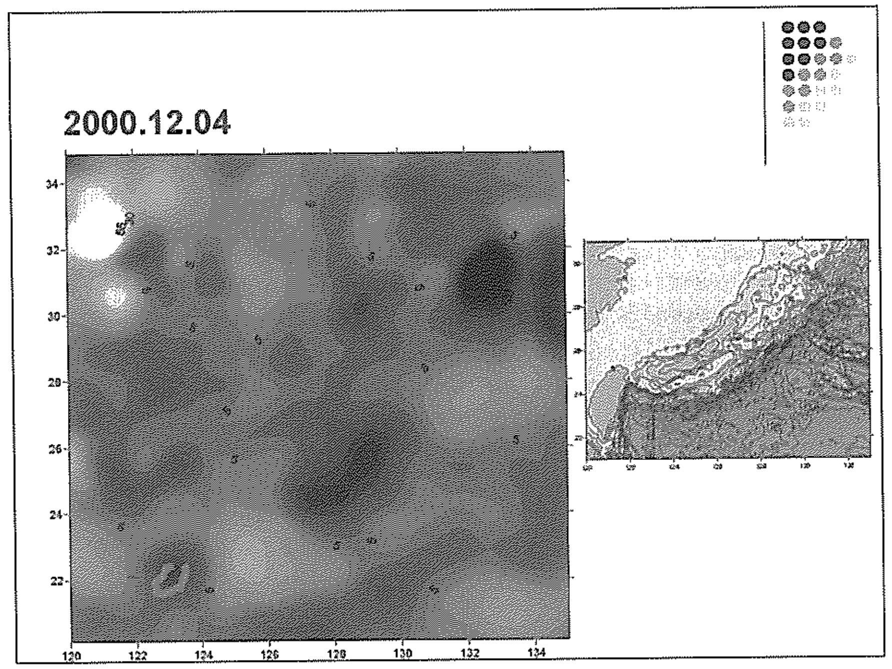
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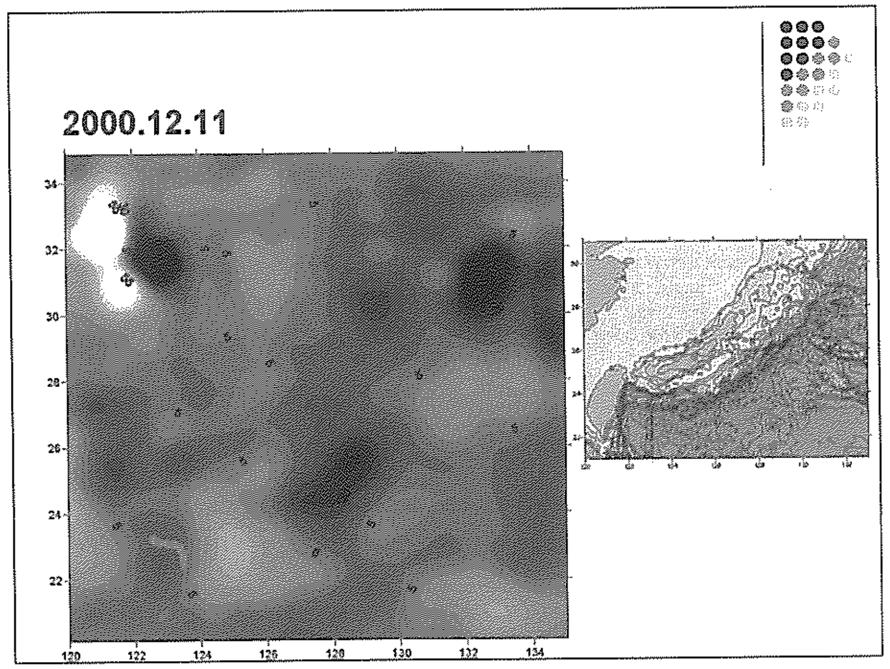
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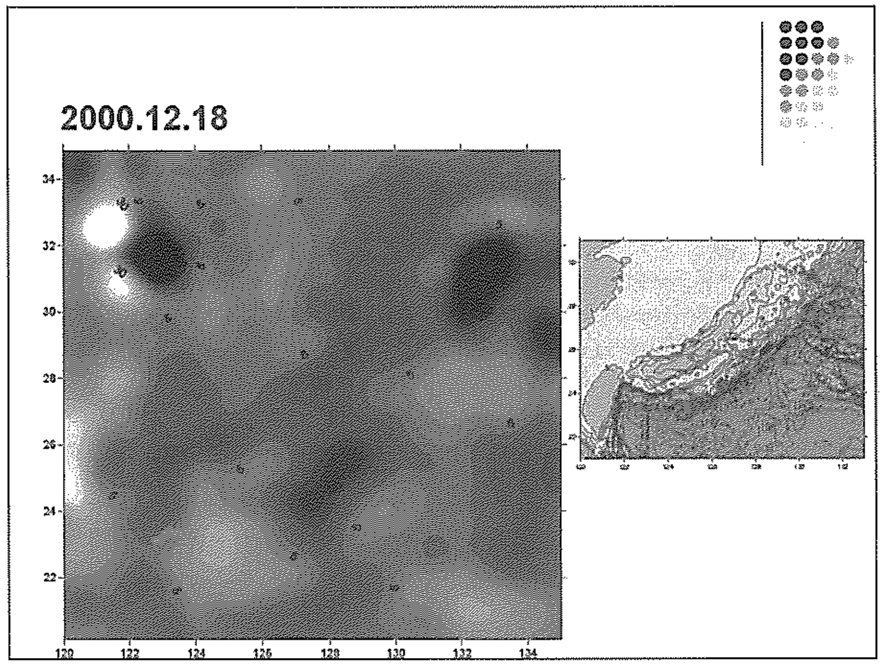
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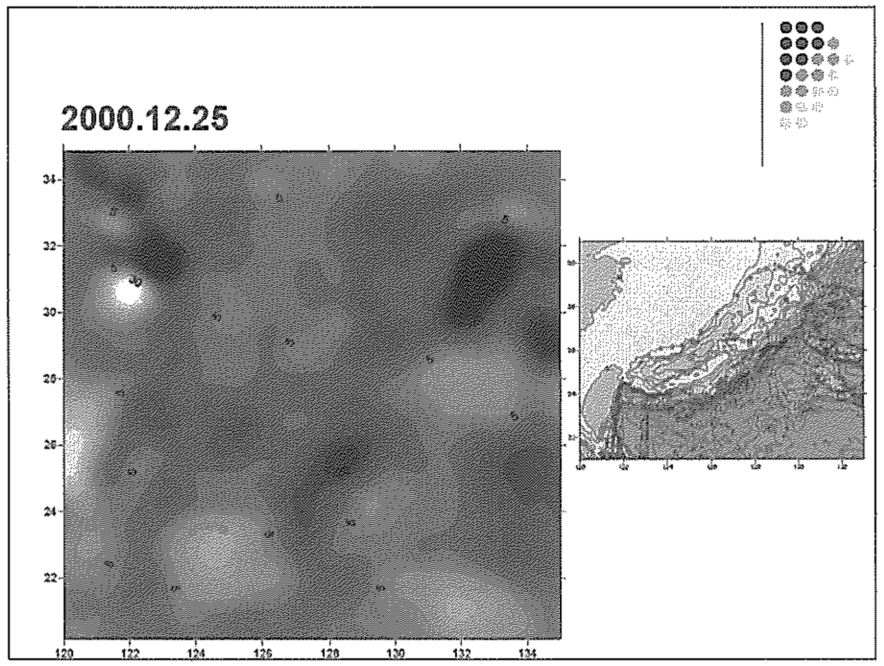
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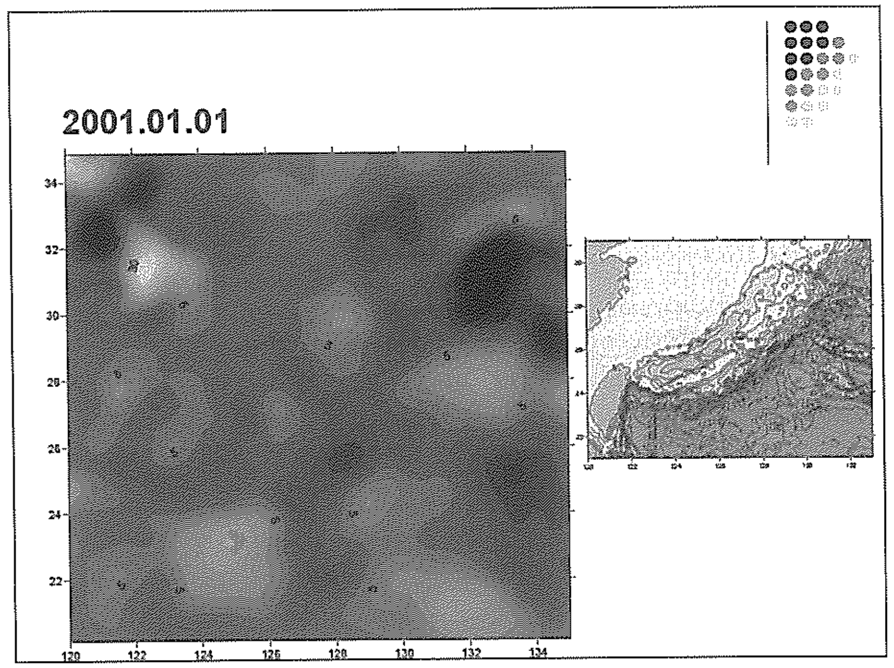
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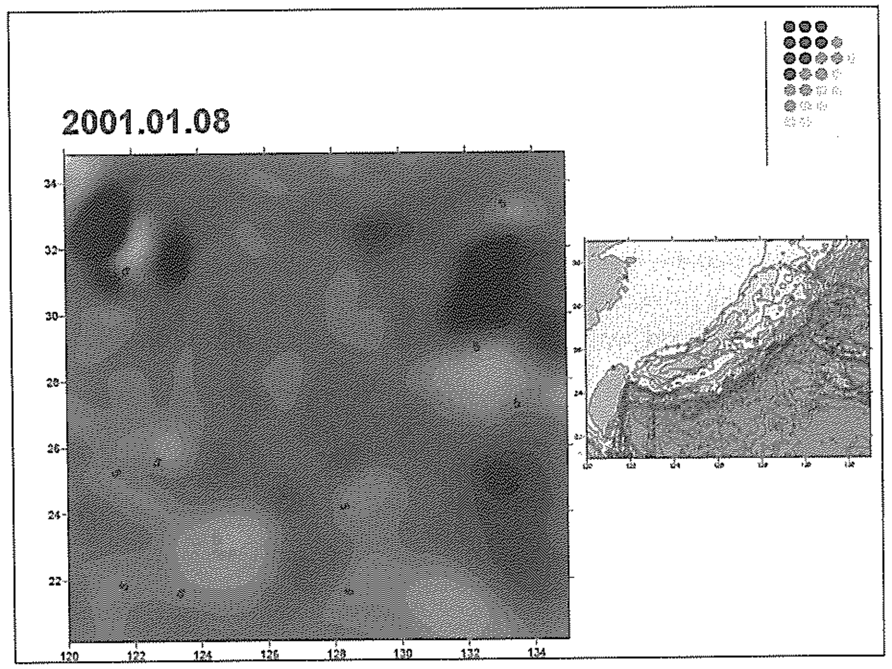
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## Concluding Remarks

- The Wavelet Rotary Spectral Analysis Method is proposed to retrieve the information of tidal and other periodic current from the drifter data. The velocity amplitude and corresponding rotary could be obtain for individual constituents at specific locations.
- The case study demonstrates that the low frequency current oscillations, induced by mesoscale eddies, is one of the dominating factors to the current field in the Western Pacific.
- The results of CR (Rotary Coefficient) analysis coincide with the fact that low anomaly rotates counter-clockwise in the northern hemisphere.
- This method could be applied to existing trajectory records of drifters to obtain the spatial variability of individual constituents.