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Transformation and Breaking of Irregular Waves on Very Gentle Slopes*

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Abstract — Based on theoretical analysis, numerical calculation, and experimental study, this paper discusses breaker indices of irregular waves, transformation of wave spectrum, characteristics and computation of breaking waves, as well as the critical beach slope under which waves will not break. Computed results are in good agreement with laboratory physical model test data and ocean wave field measurements.

Key words: *wave spectrum, beach slope, wave transformation, wave breaking, breaker indices*

1. Introduction

On a very gentle slope, wave transformation and energy dissipation are great due to the long distance of wave propagation from deep water to the coast. This phenomenon brings about some questions: How can we calculate wave transformation? Which are reasonable indices of wave breaking? And is it possible that waves may not break on a gentle slope because of wave energy loss due to bottom friction effect? From different points of view, Goda (1970), Kamphuis (1991), Li and Dong (1991, 1993, 1994) discussed these problems under one condition that the beach slope is equal or larger than 1 / 50, but studies for very gentle slopes are relatively few or not unified. For example, the results by Nelson (1983, 1987) and Goda and Morinobu (1997) are contrary. To solve these problems, Li *et al.* (1999) gave some further analysis and discussion on regular wave breaking on a beach slope of $i = 1/200$. This paper continues the study on irregular wave breaking on very gentle slopes.

2. Breaking Indices of Irregular Waves

The following may be used to determine wave breaking: geometrical stability parameter (GSP), kinetic stability parameter (KSP), and dynamic stability parameter (DSP). DSP, based on the vertical acceleration of water particle motion, is difficult to determine because the value varies violently near the breaking point. KSP has a clear physical meaning, which is defined by that the ratio of the horizontal velocity of water particles on wave surface u to wave celerity C is larger than 1.0. But the value of u, which depends on wave theories adopted, is also

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difficult to calculate for irregular waves. So only GSP may be used easily for engineering purposes since it is only dependent on the ratio of wave height H_b to water depth d_b, or the parameter of wave steepness $\alpha = (H_b / L_b) / \tanh(2\pi d_b / L_b)$ at the breaking point, in which L_b is the breaking wave length. The parameters of α and H_b / d_b have been extended by Li *et al.* (1991) for irregular waves on the slope of $i > 1/50$. It is indicated by Li *et al.* (1991) that for regular waves, the breaker index $(H/d)_b$ can be determined by Goda's formula

$$\left(\frac{H}{d}\right)_b = \frac{A\left\{1-\exp\left[-1.5\frac{\pi d_b}{L_0}\left(1+1.5i^{4/3}\right)\right]\right\}}{\frac{d_b}{L_0}} \tag{1}$$

where L_0 is the wave length in deep water, i is the beach slope, and A is a constant of 0.17. Li and Dong (1993) indicate that Eq. 1 could also be used for irregular waves, but there are two modifications: (1) The value of A should be reduced to 0.15 if H_b represents the height of individual large waves or 0.12 if H_b means the significant wave height. (2) Because irregular wave length is shorter than the theoretical value of regular waves, the value of L_0 in Eq. 1 should be taken as 0.74 L_1, L_1 being wave length calculated by linear wave theory. In the present study, the parameters of α and H/d will also be used as breaker indices for very gentle slopes. Because it is difficult to determine water particle velocities in irregular waves, the physical model test is used in this research for beach slope i smaller than 1 / 100.

2.1 Physical Model Test and Conditions

A physical model test for irregular waves is conducted in the wave-current flume (69 m in length, 2.0 m in width and 1.8 m in height) in the State Key Laboratory of Coastal and Offshore Engineering at Dalian University of Technology. At one end of the flume there is a hydraulic irregular wave maker. At the other end of flume, there is a wave absorber with a 1:6 slope bottom. The tested beach slope is 1:200. The configuration of the model and the locations of wave gauges are shown in Fig. 1. Within the wave flume, there are five stations for wave height measurements. Station 1, 7 m apart from the wave maker, is used for determining the initial wave parameters. Station 2 is at the front of the gentle slope, the distance between Station 1 and Station 2 being 7 m. Station 4 is located at the end of the slope. The distances between Station 2 and Station 3, and Station 3 and Station 4 are 12 m and 11 m, respectively. Station 5 is located 3 m behind Station 4 for the measurement of broken wave height. Besides, there are two additional wave gauges near Station 3 and Station 4 to determine the irregular wave length at the locations, and the distance between the coupling gauges is 10 cm.

During the test, the peak period of wave spectrum was firstly fixed and then this energy spectrum could be modified by regulating the amplitude of wave maker motion in order to let the large individual waves breaking between Station 3 and Station 4. In addition, visual observation was simultaneously taken at both Stations 3 and 4 to record the ordinal number of breaking waves in order to compare them with the signals measured by the gauges.

The test was carried out with two kinds of water depths: 15 cm and 25 cm at Station 4 and the corresponding water depths are 20.5 cm and 30.5 cm at Station 3 and 70 cm and 80 cm at Station 1 respectively. The peak periods of wave spectrum are 1.1 s, 1.3 s, and 1.5 s in shallower water tests, and 1.2 s, 1.4 s and 1.6 s in deeper water tests.

2.2 Irregular Wave Length

Fig. 2 shows the relationship between the measured wave length L_m for individual breaking waves and the wave length calculated by linear wave theory. By use of Least Square Method (LSM), the relationship between L_m and L_l can be described as $L_m = 0.74\ L_l$, with the correlation coefficient r being 0.962, and the standard deviation σ being 0.07. This result is the same as that given by Ochi and Tsai (1983), i. e. $L_m = 0.74\ L_l$, and also very close to Li and Dong's result (1993), that is, $L_m = 0.75\ L_l$.

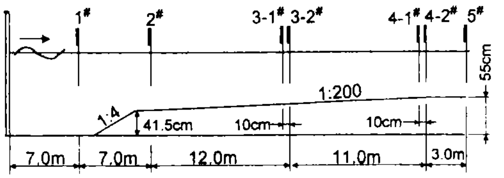

Fig. 1. Model slope and configuration of water height gage.

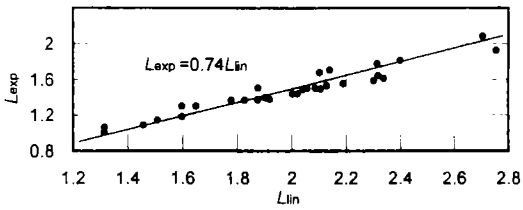

Fig. 2. Relationship between calculated and measured wave length.

2.3 Relative Wave Height H_b / d_b

The relationship between the measured relative wave height H_b / d_b for individual breaking waves and the relative water depth d_b / L_0 is shown in Fig. 3, where the wave length in deep water L_0 is given by $0.74\ gT^2 / (2\pi)$.

According to Goda's formula (Eq. 1) and by use of LSM to fit the measured data, the tested coefficient A is 0.15 and its standard deviation is 0.031. This result is the same as that from the model test on the slope of 1:50 (Li and Dong, 1993).

The relationships between the maximum and significant values of H/d i.e., H_{max} / d_b and H_s / d_b, and the relative water depth d_b / L_0 are plotted in Fig. 4. By use of LSM to fit the data, the coefficient A for each case in Eq. 1 is 0.162 and 0.11 respectively. The value ($A = 0.11$) related to significant height of H_b / d_b is slightly smaller than Kamphuis' (1991), Li and Dong's (1993) result ($A = 0.12$) which is determined experimentally for beach slopes 1:10~ 1:40 and 1:50 respectively. From the enveloping curve of the data, the value of A in this paper is still equal to 0.12.

2.4 Critical Wave Steepness Parameter α_{cr} at Wave Breaking

Based on the measured data of breaking wave height H_b and wave length L_b at Stations 3 and 4, the critical wave steepness parameter α_{cr} is calculated. The range of α_{cr} is 0.114~ 0.131 and the mean value is 0.12. The values of α_{cr} at Stations 3 and 4 are very close.

2.5 Asymmetry of Wave Profile at Breaking

As shown in Fig. 5, there are three parameters related to the asymmetric characteristics

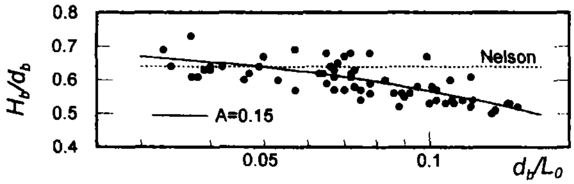

Fig. 3. Relationship between H_b/d_b and d_b/L_0 of breaking wave.

Fig. 4. Relationship between H_{max}/d_b, H_s/d_b and d_b/L_0.

of the wave profile. The first is the ratio of duration of wave front T' to that of wave back T''. The Second is the ratio of duration of wave crest $T'+T''$ to wave period T. And the third is the ratio of elevation of wave crest η_c to that of wave trough η_t. These three measured parameters at Stations 3 and 4 related to relative water depth d/L_0 are shown in Fig. 6. It is seen from Fig. 6 that the values of T'/T'' at Station 3 are from 0.81 to 0.94, while at Station 4 from 0.79 to 0.93; the values of $(T'+T'')/T$ at Station 3 are in the range of 0.28~0.42, and at Station 4 from 0.26 to 0.42; the values of η_c/η_t at Station 3 are from 1.40 to 1.70, and at Station 4 from 1.38 to 2.0. This means that the difference in wave profile asymmetry between Stations 3 and 4 is not dominant, and that the effect of relative water depth d/L_0 on wave profile asymmetry is not large. Moreover, the asymmetry of irregular wave profile is much weaker than that of regular wave profile, as reported by Li *et al.* (1999).

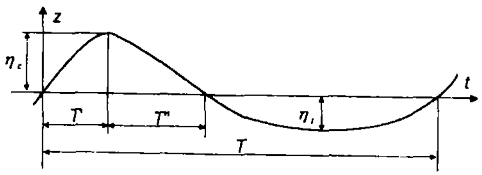

Fig. 5. Asymmetry of wave profile.

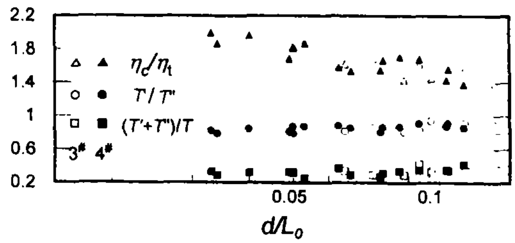

Fig. 6. Asymmetry of wave profile for irregular waves.

2.6 Recommendation of Irregular Wave Breaker Indices for Very Gentle Slopes

Based on the above mentioned experimental data and the analysis of collected data of irregular wave breaking on various beach slopes, this paper recommends the following irregular wave breaker indices for very gentle slopes:

(1) The breaker index of relative wave height H/d is suitable for irregular waves on very gentle slopes, and Goda's formula can still be used. For individual breaking large waves, coefficient A in Goda's formula is equal to 0.15 for the two tested slopes — 1:50 and 1:200. From Goda's formula, it is clear that when beach slope i is smaller than 1:100, the value of beach slope i no longer affects the breaker index H/d greatly. So it is reasonable to recommend $A=0.15$ as the coefficient in Goda's formula for various very gentle slopes, which is also

a conservative value. For a significant wave height, $A = 0.12$ may be used as a conservative coefficient.

(2) The length of irregular waves is shorter than the value calculated by linear wave theory, or, the length of irregular waves near breaking points is 0.74 times of that calculated by linear wave theory.

3. Calculation of Transformation of Irregular Waves Before Breaking

The calculation of transformation of irregular waves (spectrum) can be conducted by means of linear superposition of the transformation of wave components, whilst the computation of transformation of wave components in a spectrum may be operated by the method used for regular wave transformation. Based on the principle of conservation of wave action flux, and in consideration of bottom energy dissipation and the existence of currents, the conservation of wave action flux of wave component i in a spectrum can be expressed as follows (Li, 1989)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[\frac{S(\omega_i)\,\mathrm{d}\omega_i}{\omega_{ri}}\left(C_{gi}(\omega_i)+U\right)\right]+\frac{E_d(\omega_i)-\langle\tau(\omega_i)\rangle U}{\omega_{ri}}=0 \tag{2}$$

in which S is the wave spectrum density, ω_r is the wave angular frequency relative to current, C_{gr} is the relative wave group velocity in current, U is the current celerity, E_d is the total wave energy dissipation during wave propagation, τ is the bottom frictional stress, $\langle\ \rangle$ represents the average value in one wave period, and $\langle\tau\rangle U$ is the energy loss due to the action of current. The bottom frictional stress can be obtained by

$$\tau=\rho f_{wc}\,|U+u_w|\,(U+u_w) \tag{3}$$

in which f_{wc} is the bottom friction factor in wave-current field, and u_w is the wave particle velocity at bottom. The energy dissipation term can be expressed by the following equation

$$E_d-\langle\tau\rangle U=f_{wc}A \tag{4}$$

where term A is

$$A=\begin{cases}\rho U^3\alpha^{-2}, & |\alpha|>1\\ \dfrac{\rho U^3\left[3|\alpha|\cos^{-1}(1-2\alpha^2)+2(\alpha^2+2)\sqrt{1-\alpha^2}\right]|\alpha|^{-3}}{3\pi}, & |\alpha|\leqslant 1\end{cases} \tag{5}$$

in which $\alpha = U/u_w$. Then Eq. 2 can be rewritten as

$$S_2(\omega_i)\frac{C_{gr2i}+U_2}{\omega_{r2i}}-S_1(\omega_i)\frac{C_{gr1i}+U_1}{\omega_{r1i}}+\int_{x_1}^{x_2}\frac{Q(\omega_i)}{\rho g\omega_{ri}}\,\mathrm{d}x=0 \tag{6}$$

where subscript 1 denotes related values at x_1, and subscript 2 represents those at x_2; for waves propagating from x_1 to x_2, $Q(\omega_i)=\rho g f_{wc} A/\mathrm{d}\omega_i$.

Linearizing the term of bottom energy dissipation, taking the equivalent stress as

$$\tau^{*} = \rho f_{wc} \left(k_0 + k_1 u_w\right) \tag{7}$$

and determining coefficients k_0 and k_1 by LSM, that is, the bottom energy dissipation

$$E\left\{\left[\left(\tau^{*} - \tau\right) u_w\right]^2\right\} = \min,$$

one can obtain

$$\tau^{*} = \rho f_{wc} \left\{\left[2rZ(r) + (6 + 2r^2)\, p(r)\right]\sigma_u^2 + 4\sigma_u \left[\frac{4}{3} Z(r) + rp(r)\right] u_w\right\} \tag{8}$$

and

$$Q(\omega_i) = 4 f_{wc}\, \sigma_u \left[\frac{4}{3} Z(r) + rp(r)\right] S_{uu}(\omega_i) \tag{9}$$

where σ_u is the standard deviation of wave particle velocity, $Z(r) = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2} r^2\right)$, $r = U/\sigma_u$, $p(r) = \int_0^r Z(x)\,\mathrm{d}x$, and S_{uu} is the density of wave particle velocity spectrum, which can be determined by

$$S_{uu}(\omega_i) = |Y_{u\eta}|^2 S_{\eta\eta}(\omega_i), \tag{10}$$

and

$$Y_{u\eta} = \omega_i \frac{\cosh k(\omega_i)(z + d)}{\sinh k(\omega_i)\, d} \tag{11}$$

In the present study, the discussion is under the condition of absence of current, i. e. $U = 0$ and $r = 0$, then the equation of wave spectrum transformation can be simplified to

$$S_2(\omega_i)\frac{C_{g2}}{\omega_{2i}} - S_1(\omega_i)\frac{C_{g1}}{\omega_{1i}} + \int_{x_1}^{x_2} \frac{16 f \sigma_u S_{uu}(\omega_i)}{3\rho g \omega_i \sqrt{2\pi}}\,\mathrm{d}x = 0\,. \tag{12}$$

If the distance of wave propagation is short, energy dissipation can be ignored, so the third term on the left hand in Eq. 12 can be neglected.

4. Transformation of Irregular Waves During Breaking

The hybrid method recommended by Li and Dong (1994) can be used for the calculation of wave spectrum transformation during the propagation on a beach slope until wave breaking. This method is so powerful that high accuracy remains a high speed of computation can be attained and it is convenient for engineers to use. Before wave breaking, the calculation is conducted only in the frequency domain. Near breaking the new wave spectrum at a shallower point should be transferred into time series, and then each individual wave is checked by use of the breaker indices as mentioned above. If the value of H/d is larger than the breaker index adopted, this wave profile should be modified and reduced to the value of H_b/d_b at this point. After this modification, the modified time series should be re-transferred to the frequency domain, which may be used during calculation to the next step. The relationship between time series of wave profile and wave spectrum can be expressed as

$$\eta(t)=\sum_{i=1}^{m}\left[2S(\overline{\omega}_i)\Delta\omega\right]^{1/2}\cos(\omega_i t+\varepsilon_i) \tag{13}$$

in which $\eta(t)$ is the wave surface elevation related to still water level, assuming that the wave spectrum is combined by linear superposition of m wave components with the frequency of ω_i, $\overline{\omega}_i=(\omega_i+\omega_{i-1})/2$, $S(\omega_i)$ is the wave spectrum density of the wave component, and ε_i is the random phase angle, which distributes uniformly within 0 to 2π.

When the wave spectrum is transferred into time series $\eta(t)$, each individual wave is checked against the relative wave height H/d. If H/d is larger than the critical value H_b/d_b, this individual wave should be breaking, thus the wave height should be modified and reduced to H/H_b, and the wave profile should be modified by the following equation:

$$\eta_2(t)=\frac{H_b}{H}\eta_1(t) \tag{14}$$

where $\eta_1(t)$ is the value before modification, and $\eta_2(t)$ is the value after modification. The wave spectrum after breaking can be obtained by the Correlation Function Method. The correlation function can be expressed by

$$R(v\Delta t)=\frac{1}{N-v}\sum_{n=1}^{N-v}\eta(t_n+v\Delta)\eta(t_n) \tag{15}$$

$$\tau=v\Delta t,\qquad v=0,1,2,\dots,m$$

where N is the number of samples (in this paper $N=4096$), and Δt is the time interval (in this paper $\Delta t=0.05$ s). By integrating the Correlation Function, the rough value of wave spectrum can be obtained

$$L_n=\frac{2\Delta t}{\pi}\left[\frac{1}{2}R(0)+\sum_{v=1}^{m-1}R(v\Delta t)\cos\frac{\pi vn}{m}+\frac{1}{2}R(m\Delta t)\cos\pi n\right] \tag{16}$$

where the frequency interval is taken as

$$\begin{cases}\Delta\omega=\dfrac{\omega_m}{m}=\dfrac{\pi}{m\Delta t}\\ \omega_n=n\Delta\omega=\dfrac{\pi}{m\Delta t}\end{cases}\qquad n=0,1,2,\dots,m \tag{17}$$

The wave spectrum is smoothed by the weighting function

$$\begin{cases}D(\tau)=0.54+\dfrac{0.46\cos(\pi\tau)}{\tau_m} & \text{for } |\tau|\leqslant\tau_m\\ D(\tau)=0 & \text{for } |\tau|>\tau_m\end{cases} \tag{18}$$

$$\tau_m=m\Delta t\,.$$

The wave spectrum after smoothing is:

$$S(\omega_n) = 0.23L_{n-1} + 0.54L_n + 0.23L_{n+1}. \tag{19}$$

Therefore the wave spectrum can be obtained from time series. For high accuracy, the calculation error and its accumulation should be reduced or eliminated, so the above wave spectrum should be modified by a procedure of error correction as follows: assuming that the value of the wave spectrum before transformation is S_1, time series $\eta(t)$ from wave spectrum S_1 should be re-transferred to wave spectrum S'_1; then according to wave breaking criteria time series $\eta(t)$ is modified to $\eta'(t)$ which is the time series after wave breaking. From $\eta'(t)$, the transferred wave spectrum is S'_2. The final value of wave spectrum after error correction is S_2

$$S_2(\omega_n) = \frac{S'_2}{S'_1} S_1(\omega_n). \tag{20}$$

Then wave spectrum S_2 is used as basic data for computation of the next time step values from deeper water to shallower water.

5. Model Test Verification of Wave Spectrum Shoaling and Wave Breaking Calculation

According to the initial wave parameters measured at Station 1, the charts of wave spectrum at other stations are calculated by the above mentioned method, as shown in Fig. 7 (The solid line represents the value of calculation, and the dashed line experimental data). Also the wave spectrum parameters at both Stations 3 and 4 are shown in Tables 1 and 2. It is seen from

Table 1 Wave spectrum and breaking probability at Station 3

		I0201	I0402	I0603	I1203	I1401	I1603
	T_s (s)	1.20	1.40	1.60	1.40	1.60	1.80
T_{01}	Calculated (s)	2.54	2.77	2.96	2.65	2.93	3.10
	Measured (s)	2.55	2.57	2.83	2.77	2.72	3.02
	Error (%)	−0.39	7.78	4.60	−4.33	4.04	2.65
M_0	Calculated (cm^2/s)	3.18	4.41	4.49	6.21	8.22	10.81
	Measured (cm^2/s)	3.47	4.66	4.76	6.58	8.38	9.56
	Error (%)	−8.36	−5.36	−5.67	−5.62	−1.91	13.08
H_s	Calculated (cm)	7.13	8.12	8.48	9.97	11.47	13.15
	Measured (cm)	7.44	8.35	8.72	10.26	11.58	12.37
	Error (%)	−2.82	−2.75	−2.75	−2.83	−0.95	6.31
N	Calculated	196	147	130	188	146	123
	Measued	184	161	146	159	145	128
	Error (%)	6.52	−8.70	−10.96	−18.24	0.69	−3.90
Probability	Calculated (%)	0	0	0	1.06	8.94	9.76
	Measured (%)	1.54	1.84	2.19	1.88	7.59	4.70

these tables and figures that.

(1) The calculated results of wave spectrum parameters and the probability of wave breaking for Station 3 and 4 are very close to the measured data, and the errors are generally smaller than 10%. Especially, the errors of significant wave heights are smaller than 3%, while the errors of wave spectrum energy M_0 are smaller than 9%. The calculated values of wave breaking probability are also close to the measured data.

(2) The calculated wave spectra for Stations 3 and 4 are close to the measured results. When the significant wave periods increase, the discrepancy between the results of calculation and measurement increases slightly, which may be due to the wave non-linearity, but as shown in these tables, their effects on the characteristic parameters of wave spectrum are not important.

(3) Based on the above analysis, it is reasonable to recommend the method for the calculation of irregular wave transformation and breaking. And $A = 0.15$ in Eq. 1 as the breaker index of irregular waves can be utilized for the calculation of wave spectrum shoaling and breaking on very gentle slopes.

Table 2 Wave spectrum and breaking probability at Station 4

		I0101	I0301	I0501	I1101	I1303	I1503
	T_s (s)	1.20	1.40	1.60	1.40	1.60	1.80
T_{01}	Calculated (s)	2.61	2.89	3.05	2.73	2.97	2.98
	Measured (s)	2.45	2.73	2.90	2.72	2.88	3.12
	Error (%)	6.53	5.86	5.17	0.37	3.12	4.49
M_0	Calculated ($cm^2 \cdot s$)	2.57	2.92	2.71	6.00	9.31	9.42
	Measured ($cm^2 \cdot s$)	2.64	3.10	2.72	5.75	7.85	7.31
	Error (%)	−2.65	−3.87	−0.37	4.35	18.60	28.86
H_s	Calculated (cm)	6.42	6.84	6.58	9.80	12.20	12.28
	Measured (cm)	6.49	7.04	6.60	9.60	11.21	10.82
	Error (%)	−1.08	−2.84	−0.30	2.08	8.83	4.25
N	Calculated	185	146	124	150	151	184
	Measured	181	157	144	154	143	128
	Error (%)	2.21	−7.01	−13.89	−2.67	5.59	43.75
Probability	Calculated (%)	0	0	0	10.67	9.93	8.15
	Measured (%)	2.76	1.27	1.42	5.80	7.00	7.03

6. Analysis of Wave Spectrum Transformation and Breaking on Gentle Slopes

By use of the hybrid method for the calculation of wave spectrum shoaling and breaking and breaker indices of irregular waves, the effects of various factors (bottom slope, wave steepness, wave period and bottom friction) on the transformation and breaking of irregular waves on very gentle slope are discussed.

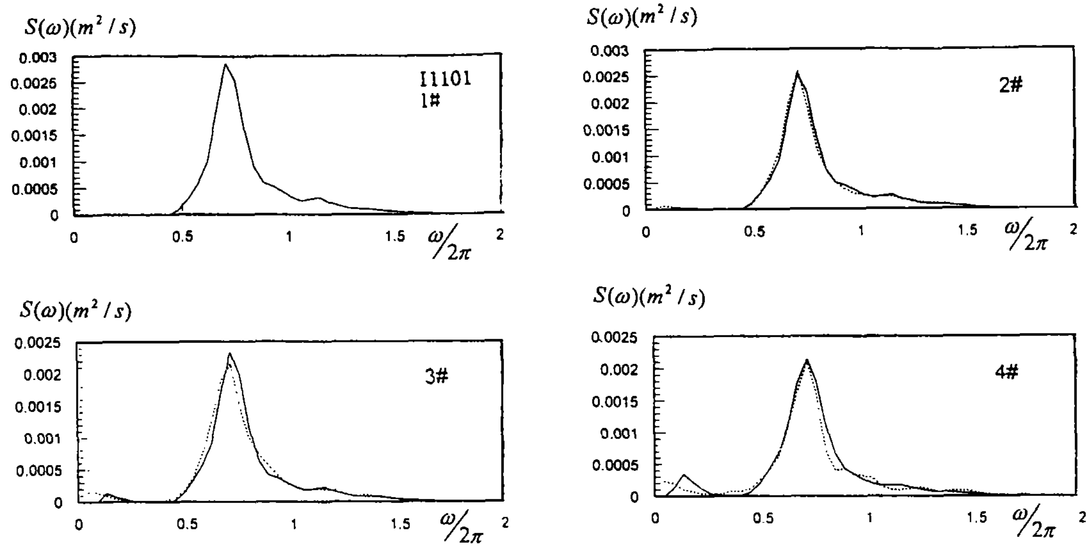

Fig. 7. Transformation of wave spectrum (N10102).

6.1 Effect of Beach Slope

The breaking point, wave parameters at this point and the critical slope on which no wave breaking occurs are calculated, as shown in Table 3. Here, the mean wave period is $\overline{T}$ = 6.0 s, the significant wave period is T_s = 6.9 s, the wave length L_0 in deep water is 2 times of the initial water depth d_0, the wave steepness is $\overline{H}_0 / L_0 = 0.026$ or $H_{s0} / L_0 = 0.0368$ ($\overline{H}_0$ is the mean wave height and H_{s0} is the significant wave height in deep water), and the bottom friction coefficient is f= 0.015. The critical slope is defined as the slope condition under which H/H_0 decreases continuously and the relative wave height H/d at each calculated point is smaller than its critical value H_b/d_b, as shown in Fig. 8. From Fig. 8, in the case of critical slope 1:1700, the variances of H/H_0 and H/d related to d/L_0 coincide with this definition; when the beach slope is larger than 1:1700, they will not coincide with this definition. In the table, d_d/L_0 means the ratio of water depth at breaking point to the initial significant wave length, H_{bs}/d_b is the value calculated by Eq. 1 (A= 0.15), the values with superscript " * " are wave lengths calculated with the wave period with respect to the smallest individual breaking wave, the values without superscript " * " are those calculated with the mean spectrum period $T_{01} = 2\pi(m_0/m_1)$, (m_0 and m_1 are the zero and first moment of wave spectrum, respectively), H_b/d_b means the ratio of the smallest breaking wave height (with superscript " * ") or the significant wave height (without superscript " * ") to the water depth, H_b/H_0 means the ratio of the smallest breaking wave (without superscript " * ") to the water depth, H_b/H_0 means the ratio of the smallest breaking wave height (with superscript " * ") or the significant wave height (without superscript " * ") to the significant wave height in deep water, and α_b is the wave steepness parameter with respect to the smallest breaking wave height (with superscript " * ") or to the significant wave height (without superscript " * ").

It can be seen from the table that, when the bed slope reduces, the critical water depth of wave breaking reduces, and the critical breaking wave height also reduces however, the relative

Table 3 The influence of different slopes

Slope	d_b/L_0	H_{bs}/d_b	H_b/d_b	H_b/H_0	α_b	n_b/N (%)
1/500	0.055	0.581*	0.675*	1.023*	0.118*	0.556
		0.549	0.471	0.714	0.087	
1/1000	0.036	0.600*	0.672*	0.667*	0.114*	0.976
		0.562	0.430	0.426	0.078	
1/1600	0.020	0.602*	0.674*	0.371*	0.114*	0.755
		0.569	0.391	0.215	0.070	

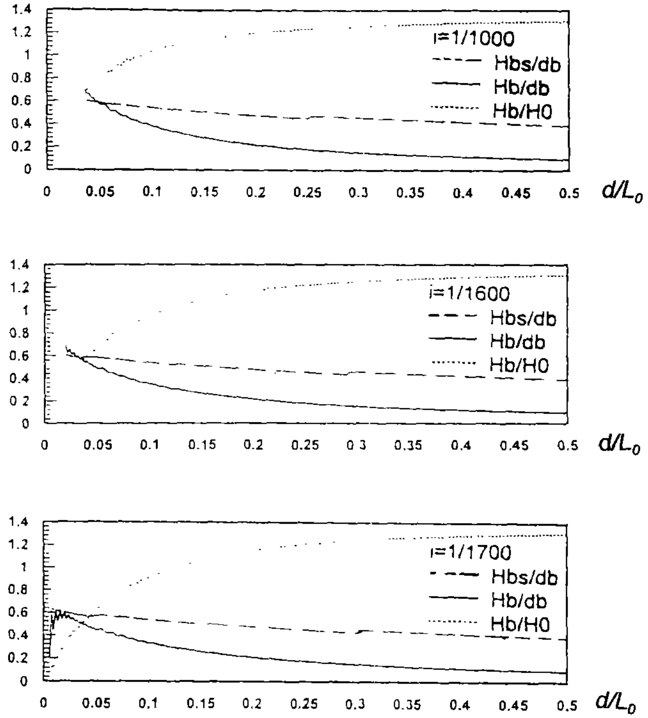

Fig. 8. Change of H/H_0 and H/d before and at critical slope.

critical wave height H_b/d_b almost remains constant, and the variance of the critical wave steepness parameter α_b is small. Generally speaking, the statistic values are a little larger than those given by Eq. 1 ($A = 0.15$), that is, the waves are already broken. Thus, for the calculation of irregular wave breaking, one may use Eq. 1 and take $A = 0.15$. The related critical wave steepness parameter α_b is between 0.114 and 0.118 and these values are close to the data of the present test.

6.2 Effect of Incident Wave Steepness

For the mean wave period 6.0 s, significant wave period 6.9 s, bottom friction coefficient $f = 0.015$, and bottom slope $i = 1/1000$, the critical wave parameters during breaking under various wave steepnesses are shown in Table 4.

Table 4 indicates that the transformation and breaking of irregular waves on the beach slope are different when the initial wave steepnesses are different. For the same beach slope, when the initial wave steepness H_0/L_0 increases, the critical water depth of wave breaking d_b increases, the relative critical wave height H_b/d_b decreases, and the critical wave height H_b/H_0 increases slightly, but the critical wave steepness parameter α_b almost keeps constant. Additionally, the critical slope i_b under which no wave breaking occurs also becomes smaller.

Table 4 Influence of incident wave steepness H_0/L_0

H_0/L_0	d_b/L_0	H_{bs}/d_b	H_b/d_b	H_b/H_0	α_b	n_b/N (%)	i_b
0.015	0.020	0.627*	0.708*	0.656*	0.117*	0.488	1/1300
		0.596	0.434	0.402	0.074		
0.026	0.036	0.600*	0.664*	0.666*	0.114*	0.976	1/1700
		0.562	0.430	0.426	0.078		
0.041	0.075	0.556*	0.637*	0.836*	0.116*	1.064	1/2200
		0.516	0.427	0.561	0.085		

6.3 Effect of Wave Period

Table 5 shows the transformation of irregular waves and the wave parameters after breaking with respect to various significant wave periods for initial mean wave steepness $H_0/L_0 = 0.041$, beach slope $i = 1/1000$ and bottom friction coefficient $f = 0.015$.

It can be seen from Table 5 that when incident wave periods vary, the wave parameters change during irregular wave breaking. When the incident significant wave period increases, the critical water depth of wave breaking becomes shallower and the critical wave height becomes smaller, but the critical relative wave height H_b/d_b and the critical wave steepness parameter α almost remain unchanged. In the meantime, the critical beach slope under which no wave bre aking occurs becomes slightly larger.

Li *et al.* (1999) pointed out that in the case of regular waves, the variance of wave period did not affect the calculated results. The cause of this difference is that for irregular waves there are various wave components of different frequencies, and the wave shoaling and bottom energy loss of various components are different. In Fig. 9, one may find, for example, that in shallower water the significant wave period becomes smaller. It means that when the significant wave period becomes larger, the long wave components increase and, during wave shoaling, their energy dissipation is also larger, so their critical water depth reduces and the critical wave height also becomes smaller.

6.4 Effect of Bottom Friction Coefficient

The wave parameters during irregular wave breaking for various bottom friction coefficients are shown in Table 6. Here, significant wave period $T_0 = 6.9$ s, initial mean wave steepness $H_0/L_0 = 0.026$ and bottom slope $i = 1/1000$.

Table 6 indicates that when bottom friction coefficient increases, the increase of bottom energy dissipation during wave shoaling causes the wave height decay greatly and the breaking point moves to shallower water. But the critical relative wave height H_b/d_b and the critical wave steepness parameter α_b remain almost constant, and in the meantime, the critical slope under which no wave breaking occurs becomes larger.

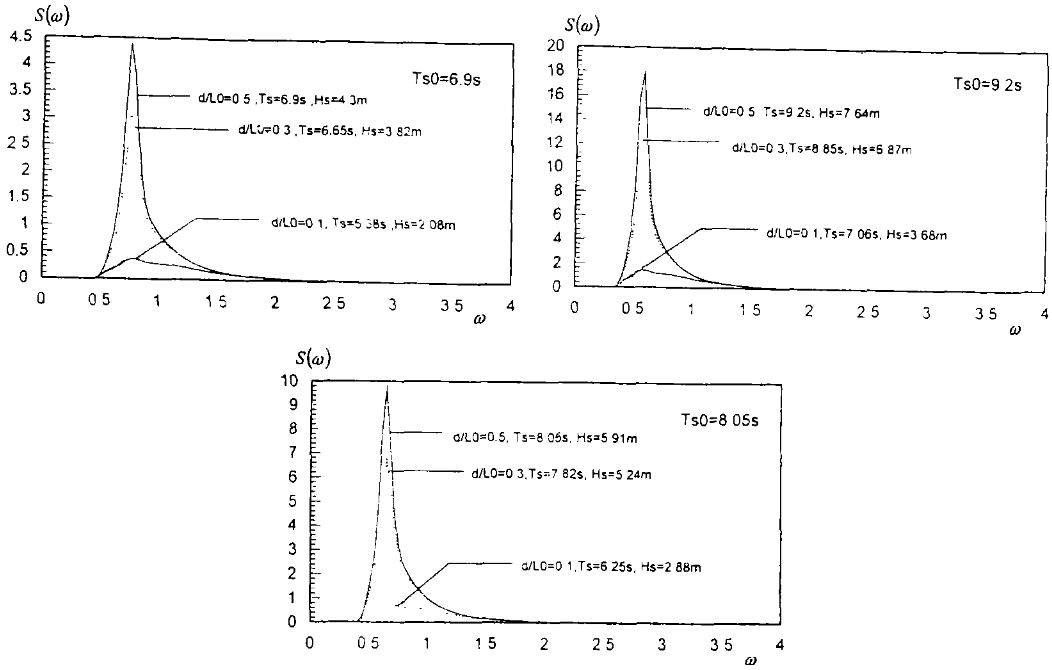

Fig. 9. Transformation of wave spectrum of various characteristic wave periods.

Table 5 Influence of incident wave period T_0

T_0 (s)	d_b/L_0	H_{bs}/d_b	H_b/d_b	H_b/H_0	α_b	n_b/N (%)	i_b
6.9	0.075	0.556*	0.637*	0.836*	0.116*	1.064	1/2200
		0.516	0.427	0.561	0.085		
8.05	0.055	0.562*	0.640*	0.614*	0.115*	2.260	1/2100
		0.527	0.458	0.439	0.089		
9.2	0.045	0.559*	0.637*	0.501*	0.116*	3.703	1/2000
		0.525	0.465	0.366	0.091		

6.5 Comparison with Results of Regular Waves

The effects of various factors on regular wave transformation and breaking are given by Li *et al.* (1999). Correspondingly, the effects under the condition of irregular waves are analyzed in the present study. The calculating conditions are the same: the wave height and period for regular waves are equivalent to the mean wave height and mean wave period for irregular waves, and the wave steepnesses of regular waves 0.03, 0.05 and 0.08 are equivalent to the initial wave steepnesses of irregular waves $\overline{H}_0/L_{s0}$ (the ratio of mean wave height to significant wave length) with the values of 0.015, 0.026 and 0.041, respectively. By comparing these results one may find that from the viewpoint of qualitative analysis they are mostly very similar and from the viewpoint of quantitative analysis they are different because of different breaker indices and different wave components. The detailed comparison is given below.

Table 6 Influence of friction coefficient

f (s)	d_b/L_0	H_{bs}/d_b	H_b/d_b	H_b/H_0	α_b	n_b/N (%)	i_b
0.0	0.075	0.571*	0.675*	1.358*	0.117*	0.654	Non
		0.541	0.498	1.028	0.093		
0.01	0.048	0.588*	0.672*	0.888*	0.116*	0.550	1/2700
		0.555	0.451	0.596	0.082		
0.015	0.036	0.600*	0.664*	0.666*	0.114*	0.976	1/1700
		0.562	0.430	0.426	0.078		

6.5.1 *Qualitative Analysis*

— The effect of beach slope on the results of regular and irregular waves is very similar. When the beach slope becomes gentler, both the critical water depth d_b and the critical wave height H_b become smaller, but the critical relative wave height H_b/d_b and the critical wave steepness parameter α_b remain almost unchanged.

— The effect of bottom friction on the results is very similar. When the bottom friction coefficient increases, both the critical water depth and the critical wave height decrease but the critical relative wave height H_b/d_b and the critical wave steepness parameter α_b remain almost constant, and at the same time, the critical slope becomes steeper.

— The effect of incident wave steepness on the results is similar in some aspects but different in other aspects. The similarity is that when wave steepness increases, both the critical water depth of breaking and the critical wave height increase, also the critical beach slope becomes gentler. The difference is that when wave steepness increases, the relative wave height H_b/d_b of regular waves remains unchanged but the corresponding value of irregular waves becomes slightly smaller.

— The variance of wave period has almost no influence on the results of regular waves. But for irregular waves, when the significant wave period increases, the transformation and energy dissipation in shallower water become more obvious and faster.

6.5.2 *Quantitative Analysis*

— Under the same condition the relative critical water depth d_b/L_0 of regular waves is smaller than that of irregular waves because the breaker indices for regular waves are larger than that those for irregular waves. But the critical relative wave height H_b/d_b is larger than that of irregular waves. For example, under the condition of $f = 0.015$ and $i = 1/500$, if the wave steepness of regular waves H_0/L_0 is 0.08, the critical values are $H_b/d_b = 0.760$ and $d_b/L_0 = 0.038$ respectively, while under the same condition, for irregular waves, if $\overline{H}_0/L_{s0} = 0.041$, the relative values are $H_b/d_b = 0.616$ and $d_b/L_0 = 0.095$ respectively.

— Wave energy dissipation of irregular waves is different from that of regular waves because irregular waves are composed of various wave components. Calculation shows that under the same condition of calculation (the same mean wave height and mean wave period), energy dissipation of irregular waves due to bottom effect is less than that of regular waves. This result together with the difference of breaking indices between regular waves and irregular waves makes that the critical slope for irregular waves is smaller than 1 / 1000, but for regular waves it is larger than 1 / 1000. In nature, waves are usually irregular and random, so it is reasonable and safe to use the results of irregular waves suggested in the present paper.

7. Comparison of Calculated Results with Field Measurement

The field wave measurement was conducted in the northern region to the Taichun Harbour by the Ocean Monitoring Center of Cheng Kung University. The instruments for wave measurement were ultrasonic wave gauges. There were two station: Station A was located at water depth −6.0 m, and at this station there were four wave gauges for directional wave spectrum measurement, the configuration of which is shown in Fig. 10; Station B was located at water depth −3.0 m, with only one wave gauge. In this sand beach region, the underwater contour lines are very uniform and straight, and the direction of sea bottom contour lines is approximately N35° E. The current velocity in this area is small, so only wave shoaling and refraction are considered. The distance between Stations A and B is 346.2 m, and the beach slope is about 1 / 120. The wind and wave data collected during July and September, 1993 are used for the present analysis and calculation. Data analysis shows that the data from No. 3 wave gauge at Station A could not be used for this analysis, so the directional propagation of wave spectrum at Station A is calculated with the data from No. 1, 2 and 4 wave gauges. The wave spectra from three wave gages are very close to each other, so their mean value of wave spectrum and the directional propagation could be used as the initial values of wave shoaling and

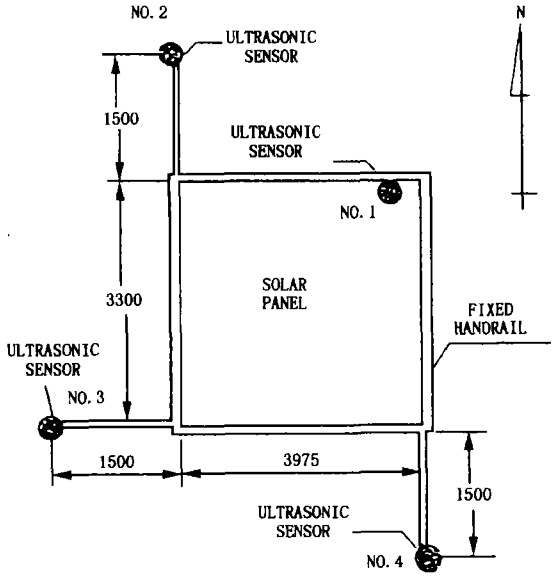

Fig. 10. Configuration of wave height gage at Station A.

Table 7 Comparison of calculated and measured data

No.	H_{s0} (m)	H_s (m)		Error (%)	Main wave direction (°)		Wind speed (m/s)	Wind direction (°)
		Measured	Calculated		A	B		
93091108	0.708	0.718	0.655	−8.78	25	10	0.16	62
93091112	0.593	0.586	0.562	−4.09	15	10	4.42	95
93091200	0.832	0.851	0.811	−4.70	30	8	1.67	−74
93091116	1.138	1.075	0.987	−8.18	5	2	8.12	76
93091120	1.087	0.983	0.980	−0.31	25	16	2.67	69
93091208	0.784	0.784	0.735	−6.25	30	16	2.05	−142
93091212	0.628	0.646	0.596	−7.70	15	10	4.92	−138
93091000	0.825	0.891	0.826	−7.29	−5	0	4.72	43
93091016	1.143	1.038	0.987	−4.91	15	10	9.29	64
93072200	0.627	0.526	0.556	5.70	−5	−2	2.31	17
93072300	0.652	0.582	0.547	−6.01	35	27	1.09	−69
93072312	0.700	0.654	0.587	−10.24	55	34	4.30	79

refraction calculation. The angles between the main wave direction and the normal direction of sea bed contour lines are between 15° and 55°, so in the calculation of wave shoaling and refraction, only the wave components propagating towards the beach are taken into account and the wave components propagating towards deep water are ignored. Experience shows that when the number of wave gauges in wave array configuration is not large (for example, three gauges), the main directional function of wave spectrum obtained from data analysis is still reliable, but the directional distribution will be wider than the actual one. So according to the above calculation principles, the analytical results from three wave gauges in the shallower region will be smaller than the real values.

Based on the above mentioned method of computation, with the wave data measured at Station A as the initial values for calculation, the shoaling and refraction of irregular waves are computed, bottom energy dissipation being considered and the bottom friction factor being $f=0.01$. The numerical results are compared with the data measured at Station B, as shown in Table 7 and in Figs. 11 and 12. H_{s0} in Table 7 represents the significant wave height measured at Station A. The main direction of wave spectrum propagation and the direction of wind are determined on the normal direction of contour lines, in which the positive angle means turning to the right hand and the negative value means turning to the left hand. Both of the main directions of wave spectrum at Stations A and B are given in Table 7. It is seen from Table 7 that the main direction of wave propagation is different from the wind direction except the data of measurement No. 9307220 and No. 93072312. So the wind effect on wave propagation will not be considered, but for the data of No. 9307220 and No. 93072312, the wind energy input should have some influence on the transfer of wave energy spectrum. The numerical results coincide with the measured data the errors being usually smaller than 10%, and the calculated values being often smaller than the observed. For the result of No. 93072312, the error is also smaller than 10% if the

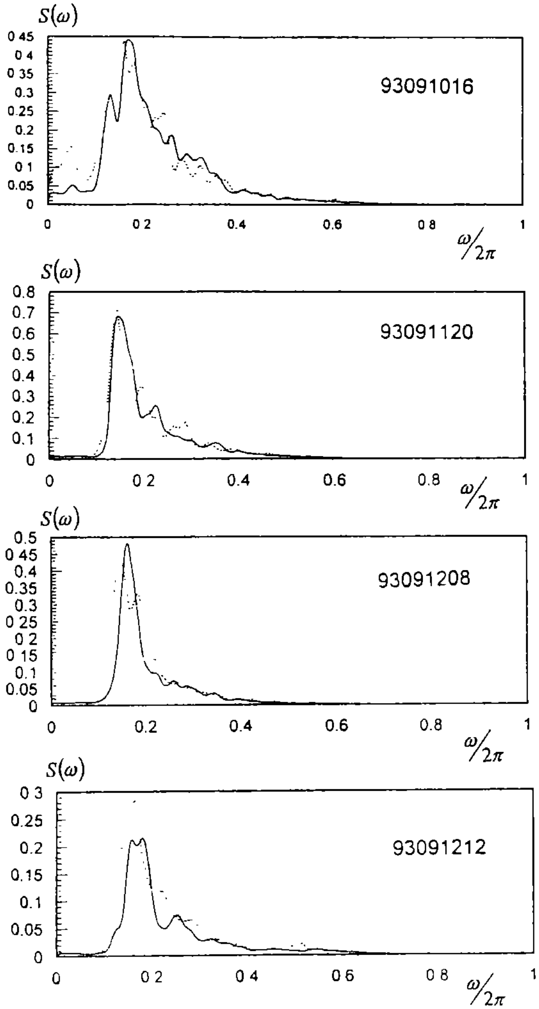

Fig. 11. Calculated and measured wave spectrum at Station B.

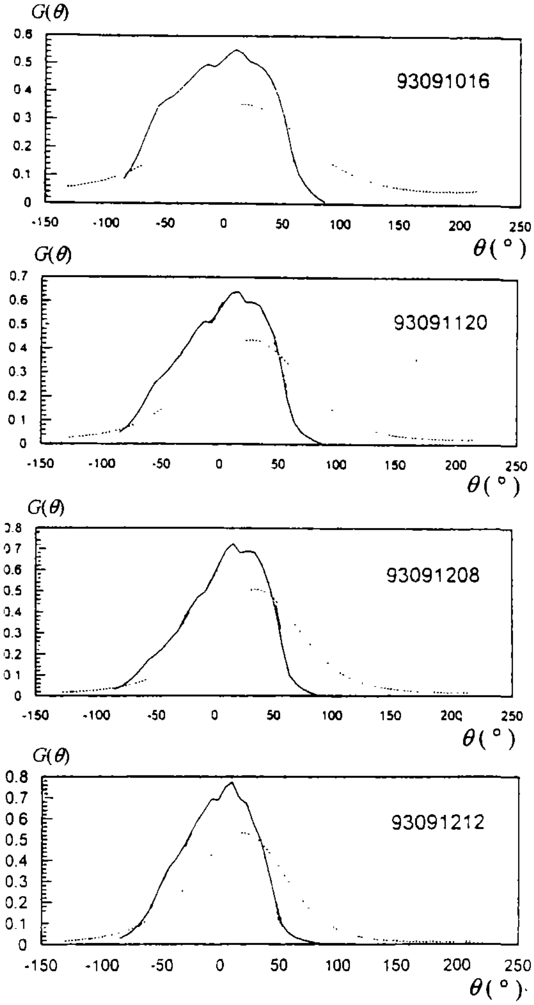

Fig. 12. Wave energy direction distribution at Station A and B.

input of wind energy that may cause wave energy increase is taken into account. As mentioned above, only the data from three wave gauges can be used for analysis of the function of directional distribution and the wave spectrum, the analyzed directional distribution may be wider than real values for the most part, and this error may lead to smaller numerical results. These results show that the suggested method may provide theoretical solutions that coincide well with actual ones. Fig. 11 gives a comparison between calculated and measured data at Station B, in which the solid lines are computed values and dashed lines represent the measured data. Fig. 12 shows the comparison between the directional distribution of wave energy at Station A (dashed lines) and at Station B (solid lines), in which the angle of 0° is the normal direction of contour lines. In calculation, only the wave energy propagating to the coast is considered, the direction varying from −90° to +90° . It is shown in Fig. 12 that the directional distribution of wave energy after refraction becomes more concentrated, and the angle between the main direction of wave propagation and the normal direction of contour lines reduces, coinciding well with the regularity of wave refraction in shallow water.

8. Conclusions

Based on the above calculation and analysis, the following can be concluded:

— The relative wave height H_b / d_b can be used as the breaker index of irregular waves propagating on a very gentle slope. The critical value of H_b / d_b can be calculated by Goda's formula (Eq. 1 in this paper) and the coefficient A should be taken as 0.15. The values of critical wave steepness parameter of large breaking waves are mostly from 0.115 to 0.12.

— The transformation and breaking of irregular waves on a very gentle slope may be calculated by the hybrid method and breaker indices suggested in this paper. The verification by both model test and field wave measurement shows that the numerical results are in good agreement with the actual ones.

— During propagation from deep water to the coast waves may not break when the beach slope is gentle enough and wave energy dissipation due to bottom effect is considered. The critical slope under which no wave breaking occurs depends mainly on the wave steepness in deep water and on the bottom friction factor, however, it is also slightly affected by wave period in deep water. Usually the critical slope for irregular waves is smaller than 1 / 1000. The critical slope becomes gentler when wave steepness in deep water increases or the bottom friction coefficient reduces.

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References

Goda, Y., 1970. A Synthesis of Breaker Indices, *Trans. of JSCE* 2 (part 2): 227~229.

Goda, Y. and Morinobu, K., 1997. Breaking Wave Height on Horizontal Bed, *Combined Aust. Coastal Engrs. and Ports Conference*, Christchurch, New Zeland, 953~958.

Kamphuis, J. W., 1991. Incipient Wave Breaking, *Coastal Engineering*, **15**, 185~200.

LI, Y. C., 1989. *Wave-Current Interaction in Ch. 7 of Wave Action on Maritime Structures*, The Press of Dalian University of Technology, 274~309.

LI, Y. C. and TENG, B., 1989. Wave Attenuation by Bottom Friction in Current, *China Ocean Engineering*, **3** (1), 59~70.

LI, Y. C., DONG, G. H. and TENG, B., 1991. Wave Breaker Indices in Finite Water Depth, *China Ocean Engineering*, **5** (1), 51~64.

LI, Y. C. and DONG, G. H., 1993. Breaker Indices of Irregular Waves on Gentle Beach, *J. of Hydrodynamics*, Ser A. **8** (1), 21~27. (in Chinese)

LI, Y. C. and DONG, G. H., 1994. A Hybrid Method for Simulation of Wave Spectrum Transformation and Breaking Phenomena on a Slope, *J. of Hydrodynamics*, Ser B. **7** (2), 76~83.

LI, Y. C., CUI, L. F., YU, Y. and TENG, B., 1999. Breaking Criteria of Regular Waves on Gentle Slopes, *China Ocean Engineering*, **13** (4), 365~374.

Nelson, R. C., 1983. Wave Heights in Depth Limited Conditions, *Civil Engineering Trans. Inst. Engrs. Aust.*, CE27 (2), 210~215.

Nelson, R. C., 1987. Design Wave Heights on Very Mild Slope —An Experimental Study, *Civil Engineering Trans. Inst. Engrs. Aust.*, CE29 (3), 157~161.

Ochi, M. K. and Tsai, C. H. 1983. Prediction of Occurrence of Breaking Wave in Deep Water, *J. of Physical Oceanography*, **13**, 2008~2019.