NOTES AND CORRESPONDENCE

An Analysis of the Characteristics of Freak Waves Using the Wavelet Transform

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ABSTRACT

Freak waves, whose causes are not yet clearly known, represent sudden emergencies of gigantic waves on the sea surface. Previous research mainly emphasized investigation of the occurrence mechanisms of freak waves through deduction; however, this paper addresses their formation based upon an analysis of observational data gained from a data buoy. The characteristics of the wave formations for the freak waves are investigated from wavelet energy spectra. Analyses of 25 cases, derived from a data buoy off the coast of Hualien, with the maximum wave height larger than twice the significant wave height for over 2 meters. We found that freak waves exhibit characteristics of maximum wave energy and its phase spectrum concentration of the corresponding instantaneous wave energy.

Key words: Freak waves, Wavelet transform, Phase spectrum
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1. INTRODUCTION

Freak waves represent a sudden emergency of gigantic waves on the sea surface. As Klinting and Sand (1987) defined that freak waves are waves with twice as large as significant wave height, and their mechanisms are still unknown. The occurrence of freak waves is mainly during typhoons and the northeasterly periods (Chien et al. 2002); they can not be ignored regarding near-shore human activities and disasters caused by ocean activities. If freak waves occur in originally tranquil sea surface, surges with gigantic strength will emerge, and damage ocean-level structures, and swamp cruising vessels (Toffoli et al. 2003). If freak waves occur in near-shore areas, the gigantic waves may engulf people having activities in these areas, cause sea water within the harbor to undulate, and endanger loading operations of the cargo vessels.

About 15 years ago, researchers all thought that the occurrence of freak waves was unpredictable (Dean 1990).

Thus, research analysis on freak waves occurrence within a short-time span was sought out and the critical issue for doing research on the freak waves is under way. Previously, the related research on the causes for freak waves was mainly on proposed methods to discriminate the possible causes for them and then to set up models entailing the discriminated causes to make calculations. Finally, they explained the calculated results of the freak waves to elaborate if freak waves will occur or not. For instance, they applied the non-linear theory (Chen 2002), or the non-linear superposition of waves (Dean 1990), or the interactions between waves and currents (White and Fornberg 1998; Liu et al. 2004) to deduce the occurrence of freak waves.

Results of the above research indicate that there are a variety of possibilities for freak wave's occurrence. This study attempts to investigate the instantaneous characteristics of energy from the on-site observational data. If the traditional Fourier Transforms are used in the wave analysis, what can be perceived is just the average energy of this section on the frequency spectrum; and the instantaneous

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energy structure, in fact, can not be viewed at all. In considering that the occurrence of freak waves is just a rather short time span phenomenon and for the sake of concretely investigating the characteristics of the freak-waves occurrence, the advantage of the wavelet transforms in time-frequency domain can be applied to analyzing the wave characteristics within this short time period. Further, this study also tries to analyze the occurrence causes of the freak waves through the wavelet energy spectra and their instantaneous energy structures.

2. METHODOLOGY

The time series data of the wave acceleration from the data buoy observations are just the very basic data source in this analysis. To obtain a wave scalogram using buoy data, the indirect method of calculating the wave spectrum from the acceleration spectrum can be adopted. The acceleration information can be transformed into the sea surface elevation information. Tucker and Pitt (2001) have pointed out the relationship that is the Double integration corresponds to multiplying the frequency spectrum by ω^{-4} ($= -\omega^{-2}$ in amplitude) between acceleration and sea surface elevation spectrum. Capobianco et al. (2002) also pointed out the wave elevations, derived from the acceleration were obtained by double integration in the Fourier domain. After that, the variations of the real water level can be perceived in order to find the occurrence time spot of freak waves.

Freak waves occur within a very short time span. For the sake of describing the instantaneous phenomena, this study applies the theory of wavelet transforms to develop a calculation procedure, i.e., to transform the wavelet energy spectrum of wave acceleration into the wavelet energy and the phase energy spectrum of the water level.

The measured wave acceleration signals from data buoy are of 10-min observations with an adopted sample frequency of 2 Hz. This paper applies the continuous wavelet transform formula and Morlet wavelet original function to analyze water level variations of the waves (Massel 2001). The Morlet wavelet function, a common wavelet function used in many applications, is chosen here to detect wave information from the acceleration signal. The continuous wavelet transform $W_A(b, a)$ of the acceleration signal $A(t)$ for a transformed wavelet function ψ is defined as Eqs. (1) and (2).

$$W_A(b, a) = \frac{1}{\sqrt{|a|}} \int_{-\infty}^{\infty} A(t) \cdot \psi^* \left(\frac{t - b}{a} \right) dt \qquad (1)$$

$$\psi_{morlet}(t) = e^{ikt} \cdot e^{-t^2/2} \qquad (2)$$

where a, representing the scale parameter, is related to the dilated frequency in the time domain. b is the shifting pa-rameter, corresponding to the position of the wavelet, as it shifts through the time domain. In Eq. (1), ψ^* is the conjugation of the wavelet original function ψ, the oscillation frequency k is generally a chosen number larger than 5, which indicates: a number smaller than 5 can not satisfy the supposition of the wavelet original function.

$W_A(b, a)$ is not the spectrum of wave signals directly, it needs to transform the parameters a and b into frequency and time domains. To obtain the spectral information from the function $W_A(b, a)$, Büssow (2007) pointed out the frequency (f) of the signal equals to the $k/(2\pi a)$. The shifting parameter b of the wavelet transform stands for the shifting distance of the wavelet function from the original location of the signal. In other words, it presents analyzed time points t of the wavelet function from the acceleration signal directly. Hence, the function $W_A(b, a)$ could be expressed as $W_A(t, f)$, which represents the acceleration spectra $W_A(f)$ from different time points t.

Similar to Eq. (1), we can obtain the wavelet scalogram of the acceleration signals $W_A(t, f)$ from the transform of acceleration time series. After being multiplied by the transform function $(2\pi f)^{-2}$, $W_A(t, f)$ will be transformed into the wavelet scalogram, $W_\eta(t, f)$ of the water level. Lang (1987) proved that the low frequency noises would have effects on the buoy data. This study filters out low frequency noises from the energy according to time-frequency characteristics of the wavelet energy spectrum for the specific data. The filtering method makes reference to the paper of Kao et al. (2003) who proposed to find a filtered demarcation between a maximum energy below the energy spectrum with a frequency of 0.02 sec^{-1} and zero energy with frequency of 0.15 sec^{-1}. After low frequency noises are filtered from $W_\eta(t, f)$, we can obtain the wavelet scalogram of sea surface elevation.

To calculate the water level time series for transform coefficient of the water level, it is necessary to go through the inverse wavelet transform. The continuous inverse wavelet transform formula of the wavelet theory is listed below and the water level time series $\eta(t)$ can be obtained through inverse wavelet transforms.

The above-mentioned method can be applied to calculate the water level time series by applying the zero crossing method to find every specific wave, and through wave statistic method to calculate the significant wave height and the maximum wave height. According to what Klinting and Sand (1987) defined for freak-wave occurrence, which is a case that will conform to the defined condition of the freak waves, it is the ratio of the maximum wave height to the significant wave height, which is larger than 2. After calculation of the water level wavelet transform coefficient, the complex function, $W_\eta(a, b)$ will be composed of variables a and b. It contains the real part, $Re(W_\eta)$, and the imaginary part, $Im(W_\eta)$. The phase angle is represented by $\tan^{-1}[Im(W_\eta)/Re(W_\eta)]$; $\sqrt{Re(W_\eta)^2 + Im(W_\eta)^2}$ is the

magnitude or modulus of energy. The calculated wavelet energy spectrum is the modus of the wavelet integration transform function. The water level wavelet energy spectrum can be obtained by multiplying the wavelet integration transform coefficient with its own conjugate and then taking the square root.

3. CHARACTERISTICS OF THE WAVELET EN-ERGY SPECTRUM OF FREAK WAVES

3.1 Wave Energy of Freak Waves During an Elapsed Time

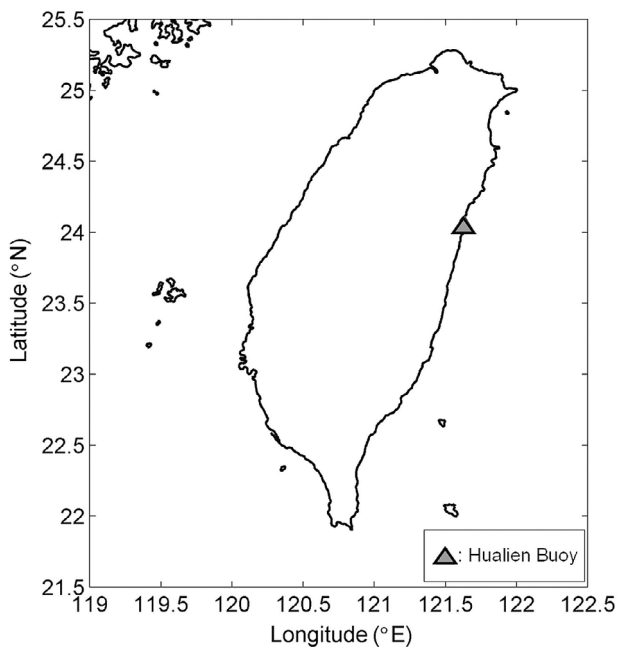According to the observed wave data from the Hualien

buoy station (Fig. 1), as shown in Table 1, there are 532 pieces of data found with a ratio of 2 between the maximum wave height and the significant wave height in 11 years (1997 - 2007). Emphasis is placed on analyzing the occurrence in regard to the different months or seasons with the freak waves for these 532 pieces of data. The analyzed outcomes are shown in Fig. 2; and it can be seen that the percentage of occurrence for monthly freak waves to occur is about 1%. Regarding the danger entailing the ocean activities, the characteristics of the specially emphasized 25 cases of data with more than 2 meters in significant wave height (Table 2) are discussed from the wavelet scalogram. This is due to the fact that the distributive characteristics of the wave energy in regard to a time- and space-domain can be perceived in the wavelet scalogram and discern instantaneous variations of the energy spectrum caused by freak

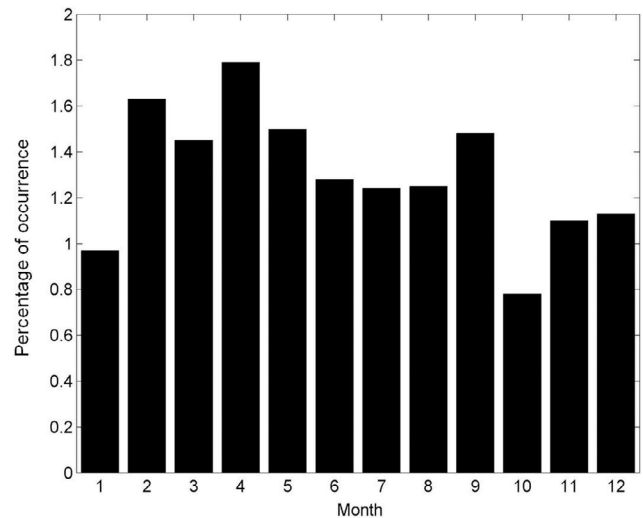Fig. 1. Location of the Hualien data buoy station.

Fig. 2. The averaged percentage of occurrence for the freak waves in months from 1997 to 2007.

Table 1. The percentage of occurrence for freak waves from 1997 to 2007.

Year	Data amounts	Freak wave case amounts	Percentage of freak wave (%)	Freak wave case amounts (for H_s > 2 m)	Percentage of freak wave [for H_s > 2 m (%)]
1997	1554	51	3.28	2	0.13
1998	2967	32	1.08	0	0
1999	2152	40	1.86	0	0
2000	3256	38	1.17	1	0.03
2001	1932	21	1.90	0	0
2002	1522	15	0.99	0	0
2003	2715	38	1.40	1	0.04
2004	3577	39	1.10	0	0
2005	4589	60	1.31	3	0.07
2006	8388	99	1.18	9	0.11
2007	8142	99	1.22	9	0.11
Total	**40794**	**532**	**1.30**	**25**	**0.06**

Table 2. Cases for the freak wave with significant wave height over 2 meters.

Case	Time	H_s (m)	H_{max}/H_s	Remark
1	1997/03/24 04:00	2.10	2.02	
2	1997/08/29 02:00	10.29	2.20	Typhoon
3	2000/04/15 14:00	2.07	2.12	
4	2003/07/22 20:00	2.57	2.03	Typhoon
5	2005/07/18 08:00	6.90	2.05	Typhoon
6	2005/08/04 10:00	2.20	2.06	Typhoon
7	2005/10/23 01:00	2.09	2.03	
8	2006/01/26 06:00	2.42	2.08	
9	2006/01/26 15:00	2.28	2.00	
10	2006/02/04 00:00	3.07	2.15	
11	2006/03/12 18:00	3.16	2.06	
12	2006/09/15 23:00	4.05	2.06	Typhoon
13	2006/10/04 20:00	2.02	2.09	
14	2006/12/06 00:00	2.16	2.03	
15	2006/12/09 17:00	2.11	2.08	
16	2006/12/10 01:00	2.02	2.04	
17	2007/01/19 06:00	2.07	2.05	
18	2007/08/08 18:00	2.1	2.06	Typhoon
19	2007/08/18 05:00	6.35	3.00	Typhoon
20	2007/08/18 06:00	6.17	2.48	Typhoon
21	2007/08/18 07:00	5.23	2.25	Typhoon
22	2007/10/29 18:00	2.03	2.06	
23	2007/11/23 05:00	2.74	2.00	
24	2007/11/27 21:00	3.66	2.01	
25	2007/11/29 21:00	2.01	2.05	

waves. This section also discusses the instantaneous characteristics for the occurring freak waves based on the occurrence timing of the freak waves.

The results of occurrence timing of the freak waves for each case are displayed in Table 3 calculated from 25 groups of freak wave cases with significant wave height larger than 2 meters. These freak waves are judged by the water level time series, the wavelet scalogram derived from occurrence time spots of the freak waves of the time series, and the maximum energy in wave energy time series. Owing to the space limitation of this paper, this article randomly selects 4 different groups of analyzed cases to display in Fig. 3. This figure shows the water level time series and the corresponding wavelet scalogram of cases 6, 7, 8 and 18. Based on the zero up-cross method, it is found that as the freak waves

appear on the freak waves positions of the water level time series (Top panel of cases 6, 7, 8 and 18 in Fig. 3), where are just very close to the occurrence of the maximum value on the wavelet scalogram time series (Bottom panel of cases 6, 7, 8 and 18 in Fig. 3). What deserves to be noted carefully within the above-mentioned 25 cases is that, within case 13 and case 22 shown in Table 3, the freak waves apparently do not occur at the maximum time spot of the value domain of the wavelet time series spectrum. Take case 22 for example; it has its occurring position for the freak wave around 284 seconds as shown in the top of Fig. 4, while the maximum wavelet scalogram time series appears around 532 seconds as shown in the middle of Fig. 4. We get the wave energy as the bottom of Fig. 4 which is the integration of the wavelet saclogram time series from the middle of Fig. 4. The bottom

Table 3. Occurring time (sec) for the freak waves in time series.

Case	Time spots for the freak waves to occur in water level time series	The corresponding time spots for the maximum in wavelet scalogram	The corresponding time spots for the maximum energy in the wave energy time series
1	34	15.5	34
2	89.5	86	89.5
3	117	118	117
4	233	232.5	233
5	217.5	220.5	217.5
6	460.5	461	460.5
7	434.5	433.5	434.5
8	255.5	254	255.5
9	354.5	353	354.5
10	1	1.5	1
11	148	151.5	148
12	255	253	255
13	386	524	386
14	313	309	313
15	251	254	251
16	537.5	526	537.5
17	258.5	257.5	258.5
18	524	521	524
19	176	177.5	176
20	503.5	508.5	503.5
21	248.5	245.5	248.5
22	284	532	284
23	20	25	20
24	253.5	249	253.5
25	306	305.5	306

of Fig. 4 indicates the spot where the energy around 284 sec. is larger than that of 532 sec. That is the reason why the frequency bands around 284 sec. are broader than that of 532 sec. in the wavelet saclogram. Therefore, after integration with respect to frequency, the energy of 284 sec. spot is larger than that of 532 sec.

Upon analyzing 25 cases of over-2-meter freak waves in significant wave height, some cases of freak waves that do not show the maximum energy on the wavelet scalogram time series. However, all data occur at the maximum wave energy through elapsed time (example shown around 284 sec in the bottom of Fig. 4). As shown in Fig. 4, by integrating wavelet scalogram in the frequency domain to obtain a

wave energy time series, it can depict variations of wave energy with respect to time, and can also analyze the timings for the maximum energy peak.

Therefore, if one wants to directly apply energy to discriminate the occurrence timings of freak waves on the wavelet scalogram time series, other methods should be included. And it is not that the large energy of one single frequency that can call freak waves into existence for sure. But from the viewpoint of the maximum wave energy through the elapsed time; currently, the analyzed freak wave always occurs on the positions where the maximum wave energy exists from the wave energy time series. An outcome is derived from the above analysis, wherein freak waves, or not,

display an apparent energy on the wavelet scalogram time series and is always there.

Except for observing the occurrence time for the energy peaks of the wave energy time series, the occurrence mechanisms of the phase characteristics of the wavelet scalogram for freak waves will be investigated in the following section.

3.2 The Instantaneous Wavelet Energy and Phase Spectrum of Freak Wave

From the analyzed results above, it indicates that if there is a freak wave this wave will always occur at the location where the maximum wave energy appears within this time span. Next, this section also discusses what a characteristic of the phase spectrum possesses individually during freak-waves occurrence.

Based on Eqs. (1) and (2), the coefficient W_A is a complex number. Thus, the coefficient of the wavelet integration can be transformed into the phase-spectrum form as phase angle $= \tan^{-1}\left[\text{Im}(W_A)/\text{Re}(W_A)\right]$. In case 22, the time span

for a freak-wave occurrence lies approximately around 284 sec in the phase spectrum (Fig. 5 top for phase spectrum at S1). This figure demonstrates that a freak wave has a more explicitly concentrated phenomenon of the instantaneous phase spectrum in the high energy region than that of its 532 sec (Fig. 5 bottom for phase spectrum at S2). In order to fully depict, within 25 cases, the differences between the phase spectrum of the freak waves and that of other no-freak waves, the standard deviations in phase angle are computed to demonstrate the status of the occurrence of the freak waves and the congregations of the phase angles for the pre-one and post-one wave of the freak waves to certify the freak-wave phenomena. The calculated results are listed in Table 4. And, apparently, the standard deviations of the spectrum phases for the occurrence of the freak waves are significantly smaller than those of the phase angles for the pre-one and post-one wave of the freak waves in most cases except for the cases of 3, 4, 7, 8, 18, and 21.

Through Figs. 6 and 7, the analyzed results of the phase angles can be elaborated. According to wave theory, the irregular waves on the sea surface can be regarded as the
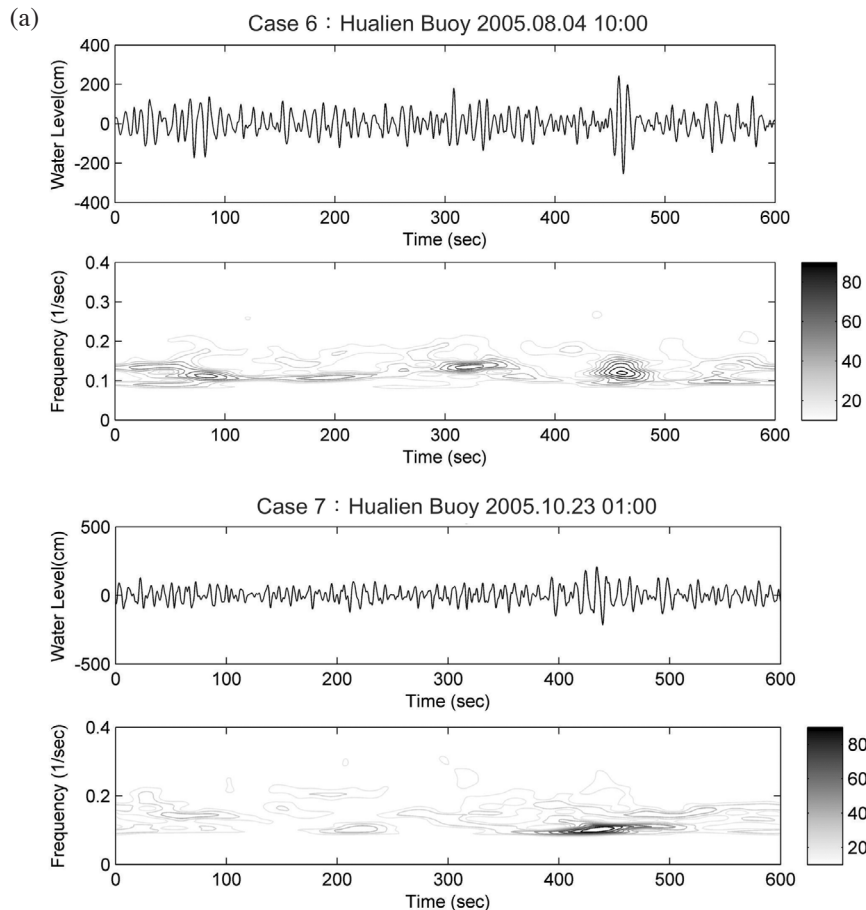

Fig. 3. (a) The water level time series (top) and the wavelet scalogram (bottom) in cases 6 and 7. (b) The water level time series (top) and the wavelet scalogram (bottom) in cases 8 and 18.
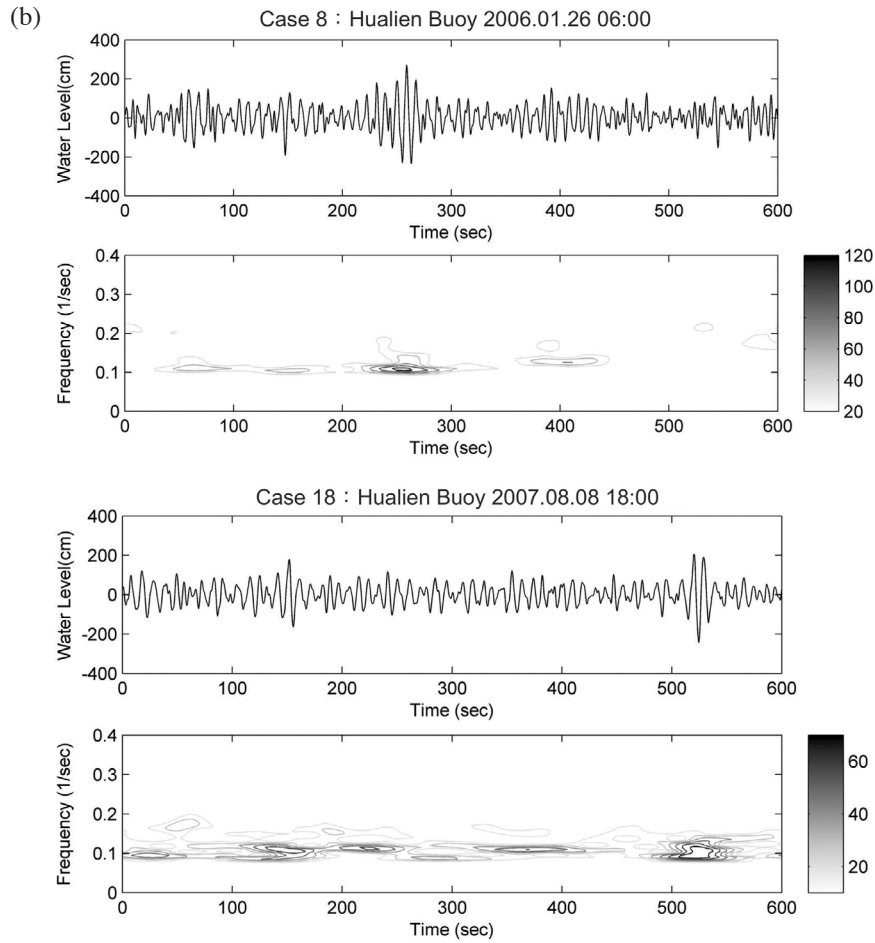
(b)

Case 8：Hualien Buoy 2006.01.26 06:00
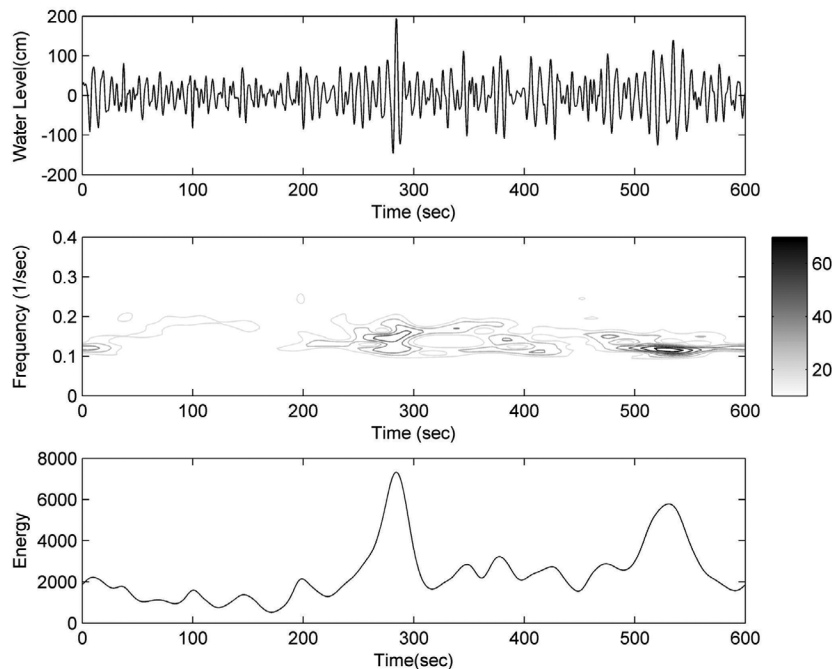
Fig. 3. (Continued)

Fig. 4. The water level time series (top), wavelet scalogram (middle), and wave energy time series (bottom) in case 22.
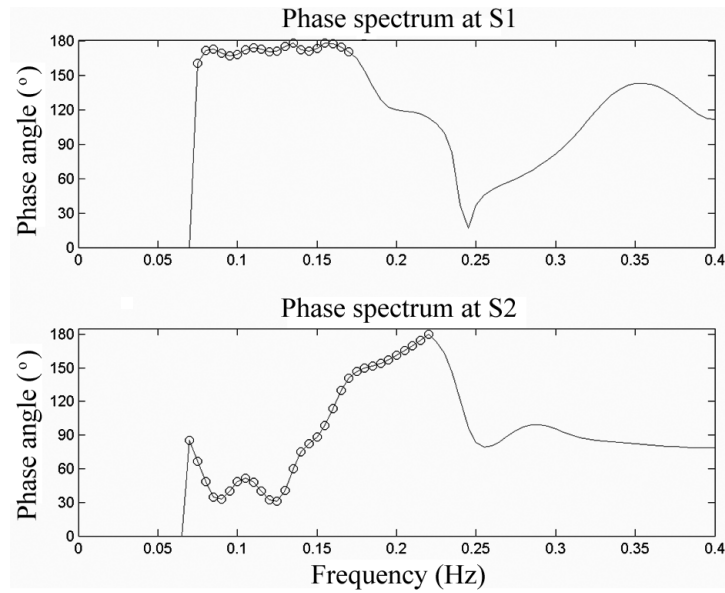
Fig. 5. The phase spectrum corresponding to case 22 at 284 sec (top, S1) and 532 sec (bottom, S2).

Table 4. The standard deviation (°) of phase spectrum for the freak waves.

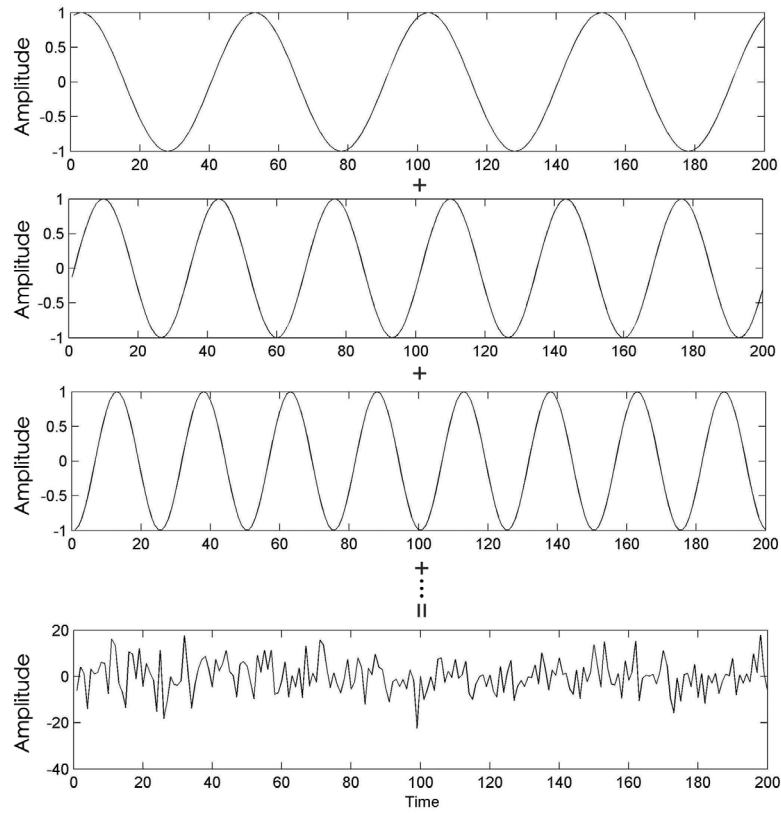| Case	The standard deviation of phase spectrum for the pre-one wave of the freak wave	The standard deviation of phase spectrum for occurrence of the freak wave	The standard deviation of phase spectrum for the post-one wave of the freak wave
1	165	36	250
2	102	22	94
3	25	31	71
4	39	58	47
5	159	32	164
6	104	52	144
7	75	45	38
8	60	72	51
9	73	18	88
10	69	27	107
11	165	36	250
12	103	44	95
13	62	42	78
14	40	24	84
15	64	46	56
16	83	26	53
17	68	26	75
18	90	50	47
19	112	16	223
20	26	26	142
21	64	86	87
22	94	26	64
23	35	25	37
24	58	24	143
25	100	21	84

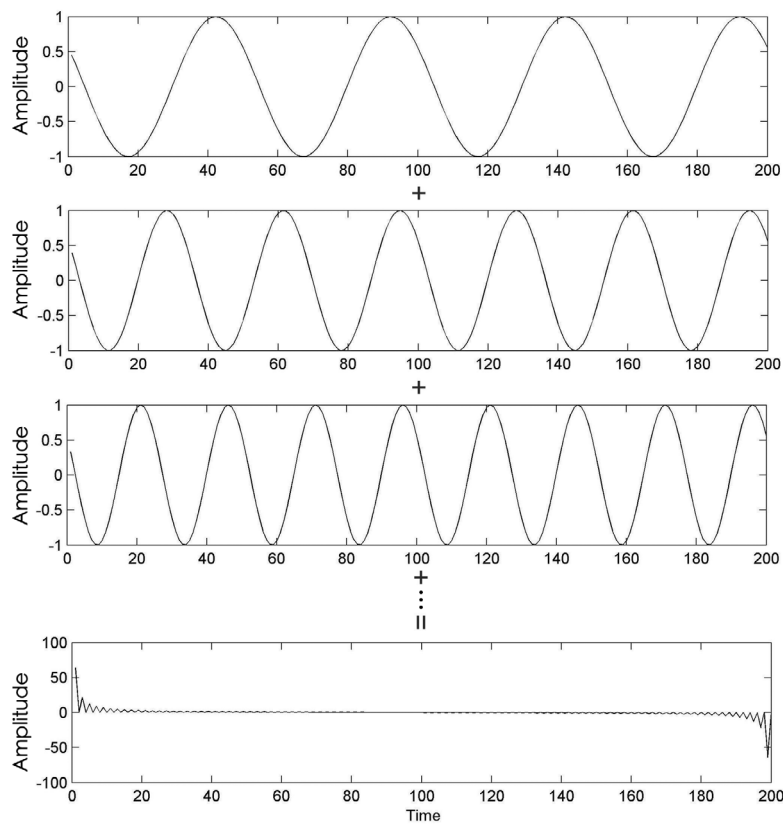Fig. 6. The superimposed results of the sine component waves under the random phase conditions.

Fig. 7. The superimposed of the sine component waves under the initial condition equal to zero.

integration results of the different amplitudes, frequencies and directional sine and cosine of the component waves. Figure 6 indicates the superimposition results of the sine waves under the conditions of the random phases, and the composite waves do represent a signal of the irregular waves. If the phase angles of the component waves in Fig. 7 are forced to be set as a fixed value, such as all phase angles of the component waves are set to zero, then the maximum value will occur at the specific spots for the superimposed composite waves. And it is concluded that a main factor for causing most freak waves is due to the superimposition of components of the similar phase angles.

There are 6 cases (case 3, 4, 7, 8, 18, and 21) in Table 4, which show that the standard deviations of the spectrum phases for the occurrence of the freak waves are not significantly larger than those of the phase angles for the pre-one and post-one wave of the freak waves in most cases. Within these 6 cases, cases of 3, 7 and 18 show that though the standard deviations of the spectrum phases for the occurrence of the freak waves are larger or smaller those of the phase angles for the pre-one and post-one wave of the freak waves, and there exist no significant differences among them. This indicates that there exist no significant differences among the phase angle standard deviations for the occurrence of the freak waves, which are larger or smaller than those of the phase angles for the pre-one and post-one wave of the freak waves in cases of 3, 7 and 18. The analyzed outcomes of cases 4, 8 and 21 indicate the standard deviations of the spectrum phases for the occurrence of the freak waves are somewhat larger than those of the phase angles for the pre-one and post-one wave of the freak waves in most cases. Thus, analyzed outcomes of the standard deviations of the phase angles for the pre-one and post-one wave of the freak waves within these 6 cases are, separately, listed in Table 5. From this table, it is found that, within these 6 cases, except for case 8, the remaining cases depict the fact that the standard deviations of the spectrum phases will be significantly enhanced for the pre-two and post-two wave of the freak waves. Taking case 3 (Fig. 8) for example, it shows the wave spectrum distribution of the occurrence for the freak waves and the pre-one, pre-two or the post-one, post-two waves of the freak waves. The analyzed outcomes can clearly depict that the phase angle distribution of the second wave prior to the occurrence of the freak wave is significantly more diverse than that of the occurrence for the freak wave and the first wave prior to the occurrence for the freak wave. It is due to being diverse of the phase angles that the waves of similar energy can not be superimposed into forming gigantic waves. In order to grasp the above-mentioned phenomena, investigation should be carried out from the aspect of characteristics of the time series of the water levels. From the cases of the time series of the water levels in Fig. 3, it appears that, within some cases, though the amplitude of pre-one or post-one wave of the occurrence of the freak wave will be smaller than that of the freak waves, the amplitudes of pre or post freak waves are still large. This indicates that waves will form a series of wave groups within duration for the pre and post the occurrence of the freak waves. And this should be the possible cause for phase standard deviations of the occurrence of the freak waves being smaller than those of the spectrum for the pre-one or post-one wave of the abnormal wave. Thus, from 25 cases selected, it is found that the characteristic phase distribution of the abnormal wave in case 8 is significantly different from that of the other 24 cases. Further. uninvestigated mechanisms for the factors of the freak waves to occur have been revealed deserving further investigation as well.

4. CONCLUSIONS

This study applies the advantageous fact that the wavelet scalogram simultaneously possesses energy distributions for time and frequency to investigate the energy characteristics during the occurrence of the freak waves. After comprehensive analysis and discussion, a few conclusions can be drawn.

Table 5. The standard deviation (°) of phase spectrum for selected cases of freak waves.

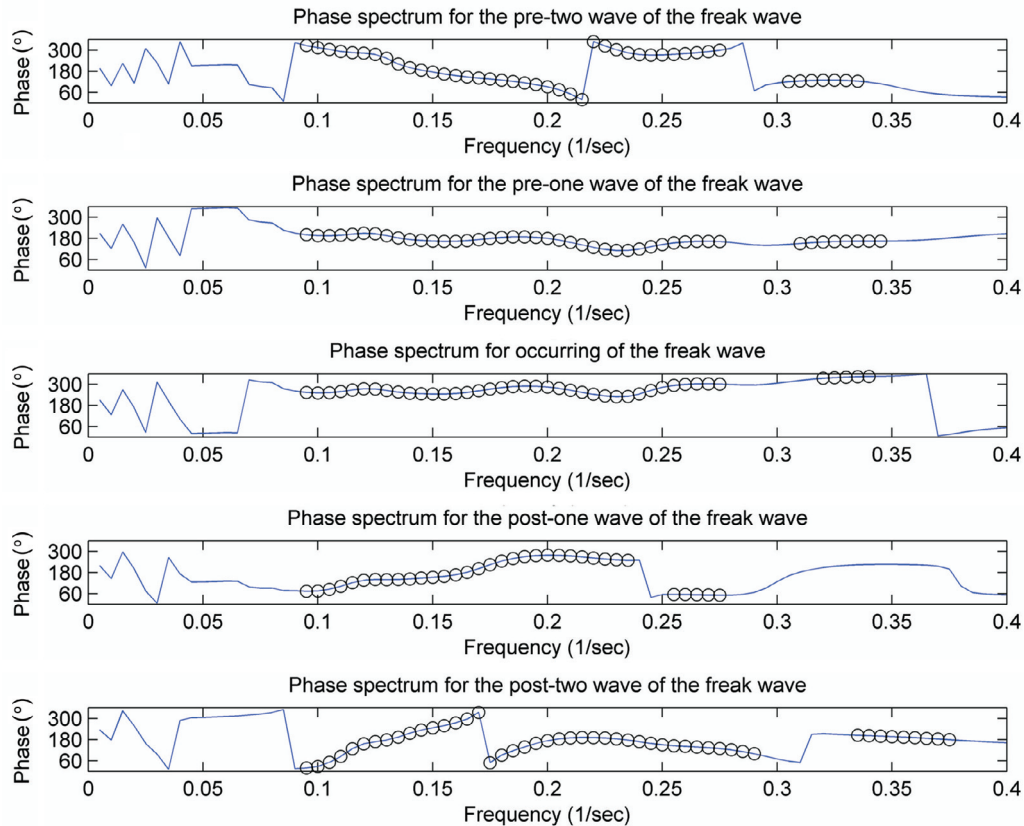| Case	The standard deviations of phase spectrum for the pre-two wave of the freak wave	The standard deviations of phase spectrum for the pre-one wave of the freak wave	The standard deviations of phase spectrum for the occurrence of the freak wave	The standard deviations of phase spectrum for the post-one wave of the freak wave	The standard deviations of phase spectrum for the post-two wave of the freak wave
3	140	25	31	71	67
4	156	39	58	47	134
7	41	75	45	38	55
8	43	60	72	51	29
18	88	90	50	47	53
21	92	64	86	87	167

Fig. 8. The phase distributions for the occurrence of the freak waves and the pre- and post- wave of freak waves (case 3).

First, the occurrence positions of the freak wave can not be directly judged by the highest point of the wavelet scalogram. Actually, it has to be obtained by the corresponding wave energy (the integration of the wavelet scalogram with respect to frequency) of the time referred to. Second, through analysis of the wavelet phase spectrum, it is deduced that as freak waves occur the component waves will lead to constructive superimposition due to similar phases. As viewed from the energy spectrum, freak waves occur with concentration of the maximum energy and phases. Finally, from the analyzed results, it can be concluded that freak waves always occur with the corresponding maximum energy, and the characteristics of the phase concentration corresponding to the peak energy.

Through study in this paper, it is perceived from the wave theory that wave components of different frequencies will possess different phase speeds; the phase will be most concentrated as freak waves occur. This can also account for the sudden emergence or disappearance phenomena of the freak waves.

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