Spectral Wave Modelling on Unstructured Grids with the WWM (Wind Wave Model) I: The deep water case

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table

Abstract

A numerical wave model was developed, which solves the (WAE) Wave Action Equation on spatially unstructured grids, with the aid of the fractional step method (Yanenko, 1971). For the integration of the WAE in spatial space we used the CNTG (Crank Nicholson Taylor Galerking) method and for the integration in spectral space the Ultimate Quickest scheme of Leonard (1992) was implemented. The new model was verified vs. measurements data at the Sargasso Sea and the Baltic Sea. Two different source term formulations for the deep water physics have been investigated and it was found that the alternative formulation of the spectral balance, can improve the hindcast of the wave kinematics essentially. Furthermore we have investigated the effect of wave attenuation of fast running waves and of waves which are running opposing to the dominant wind direction. First the effects have been quantified with the aid of numerical experiments on growth and dissipation and hereinafter we have investigated the effect of this physical mechanism on the results of the wave hindcast. We found a slightly improvement in the correlations between measured and hindcasted integral wave spectra, tough the effect was less significant in comparison to the improvements already achieved
with the new spectral balance.

1 Introduction

The newly developed model has been used in different environments; at the Sargasso Sea (Atlantic Ocean) the Baltic Sea and the Pacific Ocean around Taiwan (see Hsu et al. (2005) and Roland et al. 2005). The source functions describing the effects of wind input and white capping dissipation have been replaced in both models. For the wind input function the formulation of Makin & Stam (2002) and for the white capping dissipation function the formulation of Alves & Banner (2001) has been incorporated into the wave models. The reason for the modification of the spectral balance is that the dissipation function suggested by Komen et al. (1984) following the theoretical approach of Hasselmann (1974) leads to an underestimation of low frequency energy and an overestimation of high-frequency energy which results in an erroneous spectral shape and an underestimation of the average period (Rogers et al. (2002), Roland et al. (2004, 2005)). Moreover Holthuisen et al. (2001) and Hurdle & van Vledder (2001) have showed that the actual formulation that is used in the SWAN model may lead to erroneous dissipation rates in the case of mixed sea conditions (windsea and swell). Van der Westhuysen et al. (2005) have shown that the dissipation function of Alves & Banner reduces this effect and that it also leads to better convergence rates and better estimation of the average period.

2 Model formulation

The conservation equation describing the advection and refraction of waves due to depths and currents can be written for Cartesian coordinates as follows.

\[
\frac{\partial}{\partial t} N + \frac{\partial}{\partial x} (c_x N) + \frac{\partial}{\partial y} (c_y N) + \frac{\partial}{\partial \sigma} (c_{\sigma} N) + \frac{\partial}{\partial \theta} (c_{\theta} N) = S_{\text{total}}
\]

where \( N = N(t,x,y,\sigma,\theta) \) is the wave action density spectrum; \( t \) is the time; \( c_x \) and \( c_y \) are the wave propagation velocities in \( x \) and \( y \) space, respectively; \( c_{\sigma} \) and \( c_{\theta} \) are the wave propagation velocities in \( \sigma \) and \( \theta \) space, respectively; \( \sigma \) is the relative frequency and \( \theta \) is the wave direction. \( S_{\text{total}} \) is the source function including the energy input due to wind (\( S_{\text{in}} \)), the nonlinear interaction in deep and shallow water (\( S_{\text{nl4}} \) and \( S_{\text{nl3}} \)), the energy dissipation in deep and shallow water due to whitecapping and wave breaking (\( S_{\text{ds}} \) and \( S_{\text{br}} \)), the energy dissipation due to bottom friction (\( S_{\text{bf}} \)) and nonlinear interactions between waves and the sea floor (\( S_{\text{bg}} \)).

\[
S_{\text{total}} = S_{\text{in}} + S_{\text{nl4}} + S_{\text{ds}} + S_{\text{nl3}} + S_{\text{br}} + S_{\text{bf}} + S_{\text{bg}}
\]

The nonlinear terms \( S_{\text{nl4}} \) and \( S_{\text{nl3}} \) have been evaluated with the Discrete
Interaction Approximation (DIA, Hasselmann et al. (1981)) and the Lumped Triad Approximation (LTA, Elderberky (1999)) respectively. The dissipation formulation for bottom friction is based on the empirical JONSWAP model by Hasselmann et al. (1973) with a constant dissipation coefficient of -0.067. Bragg Scattering is not accounted in the latest version but it will be considered in the future following the approach of Ardhuin & Herbers (2002). Wave diffraction is already implemented on the foundation of the approach by Holthuijsen et al. (2005) and under testing. In the investigated simulations these effects have been neglected. The wind input function (eq. 3a) is defined following Makin & Stam, 2001 on the foundation of the work of Makin & Kudryavtsev (1999). In conditions when the waves run opposite to the wind direction the formulation by Young & Sobey (1985, eq. 3b) was used.

\[
S_{in(\sigma, \theta)} = \frac{\rho_a}{\rho_w} \cdot \min \left\{ A_w, m_p \cdot \left( 1 - m_c \left[ \frac{c_p}{28 \cdot u_w} \right]^{0.5} \right) \cdot \left( \frac{u_w}{c_p} \right)^2 \cdot \cos(\theta - \theta_w) \cdot \cos(\theta - \theta_w) \cdot \sigma \cdot N_{(\sigma, \theta)} \right\}
\]

if \( \cos(\theta - \theta_w) \cdot \cos(\theta - \theta_w) < 0 \)

\[
S_{in(\sigma, \theta)} = -c_{opp} \cdot \frac{\rho_a}{\rho_w} \cdot (a_{(\sigma, \theta)}k)^2 \left( 1 - \frac{U_w}{c} \right)^2
\]

The dissipation function (eq. 3) is based on the work of Alves & Banner. Here we used a slightly different form according to the work of Makin & Stam.

\[
S_{ds(\sigma, \theta)} = -C_{ds} \cdot \left[ \frac{B_{(\sigma)}}{Br} \right]^{p/2} \left( \frac{\alpha}{\alpha_{pm}} \right)^m \left( \frac{k}{k_0} \right)^n \cdot \sigma \cdot N_{(\sigma, \theta)}
\]

with:

\[
\alpha = \sqrt{E_{in} \cdot \overline{k}^2}
\]

\[
B_{(\sigma)} = \int_0^{2\pi} B_{(\sigma, \theta)} d\theta, B_{(\sigma, \theta)} = k_0^3 \cdot c_g \cdot \sigma^2 \cdot N_{(\sigma, \theta)}
\]

\[
p = \frac{p_0}{2} + \frac{p_0}{2} \cdot \tanh \left[ 10.0 \cdot \left( \left( \frac{B_{(\sigma)}}{Br} \right)^{1/2} - 1 \right) \right], B_{(\sigma)} > B_r,
\]

\[
p = 0, B_{(\sigma)} < B_r
\]

In the above equations \( N_{(\sigma, \theta)} \) is the action density spectrum \([m^2/Hz^2]\), \( S_{in(\sigma, \theta)} [m^2/Hz]\)
and $S_{ds}(\sigma, \theta) \text{[m}^2/\text{Hz}]$ are the rate of change in action density due to the influence of wind input and dissipative processes respectively. $B(\sigma, \theta)$ is the saturation spectrum, $\rho_a \text{[kg/m}^3\text{]}$ and $\rho_w \text{[kg/m}^3\text{]}$ are the wind and the water density, $u_* \text{[m/s]}$ and $c_p \text{[m/s]}$ is the wind induced friction velocity and the phase velocity of a certain wave component. $\theta$ and $\theta_w$ are the wave direction and the wind direction respectively and $\sigma$ is the Doppler shifted (by current in the water column) relative frequency, $k \text{[1/m]}$ and $\bar{k} \text{[1/m]}$ are the certain wave number and the average wave number of the wave spectrum, $\alpha [-]$ and $\alpha_{PM} = 4.57E-3 [-]$ are the average steepness and the Pierson-Moskowitz steepness. $C_{ds} = 2.5E-5$, $Br = 4.E-3$, $m_p = 0.045$, $m_c = 2$, $n_c = 1$, $p_0 = 8$, $m = 2$, $n = 2$, are parameters. $A_W$ controls the wave attenuation of waves that are running faster than the wind, $a(\sigma, \theta)$ is the wave amplitude spectrum and $c_{opp}$ controls the damping of the waves during opposing wind conditions. In the following we will refer to the default source term formulation in the SWAN model as C-I (combination I) and to the new source term formulation as C-III (combination III). The friction velocity $u_*$ can be evaluated with the aid of the friction law by Wu (1982) or with the sea drag parameterization by Makin 2003, which incorporates the wind-over-waves coupling theory (WOWC). The formulation of Makin is based on a resistance law, which relates sea drag to the sea state. In this study we have used the formulation of Makin for the estimation of the friction velocity.

**Test calculations**

In order to investigate the effects of wave attenuation in following and opposing wind conditions we have constructed two cases where the sensitivity of the parameters in the new source term formulation that are accounting for these two effects are evaluated.

First we have investigated the effect of wave attenuation for fast running waves which is controlled in the wind input formulation by the parameter $A_W$. The numerical experiment was carried out in a rectangular domain with a width of 1000km and a length of 5000km (see Fig. 1). The wind is set to blow parallel to the longer side of the rectangle but only for the first 1000km the following 4000km the wind is set to zero. The waves which are generated in the wind field are running in a calm region where they are traveling than with head wind. We have used different coefficients for the attenuation parameter. Additionally we have investigated the effect of whitecapping in the calm region. The differences in wave height, plotted in Fig 2, are made to a reference simulation where in the calm region no Whitecapping and no Wave Attenuation of fast running waves takes place. It can be seen that most of the energy is dissipated by the whitecapping function at the beginning of the calm region. The damping due to wave attenuations solely is only about 0.3m (for $A_W = -200$). Therefore it can be said that in comparison to the damping already done by the whitecapping function the influence of the wave attenuation like suggested by Makin & Stam is rather small.
Figure 1 Calculation Domain and observation points and cross section.

Figure 2 Influence of the attenuation parameter and the whitecapping function on the wave height of waves which are running into a calm region. Differences between calculation runs with and without the effect of wave attenuation and whitecapping in the calm region when the waves have headwind. The results are plotted along a cross section in the middle of the model domain. (Schönfeld (2005))
In the second experiment our intention was to see the effect of wave damping in opposing wind conditions due to the formulation of Young & Sobey. Therefore we used the same calculation grid like for the attenuation case but with different wind boundary conditions. The wind blows for this experiment with 20m/s from left to right 48hrs and turns then from right to left within 2hrs and stays constant for the following 48hrs. When the wind turns for 180° the waves are running against the new wind direction and are damped by the formulation of Young & Sobey. The coefficients for $c_{opp}$ that are suggested by Young & Sobey ($c_{opp} = -0.7 \pm 0.2$) lead to very small damping rates. We have used for our experiments much higher damping coefficients to have clear effect on the results. The damping of the waves when the wind turns is shown in Fig. 3, for five different positions along the modeling region. The waves are damped up to 2.5m in comparison to the case with no damping in opposing wind conditions. The maximum damping occurs in the first 10h hours when the wind turns around, than these wave components that are running against the wind are attenuated while the new waves growth in the opposite direction. Due to the damping of the opposing waves the wind wave spectra can reorganize faster in the new wind direction.

**The Sargasso Sea**

For the case of the Sargasso Sea we have simulated the September period in the year 2004 which is characterized by heavy storms and hurricanes. Starting with Hurricane “Francis” followed by “Ivan” and “Jeanne” this time period was one of the most violent ones in past years in this region. The wind input for the numerical simulations was obtained from the Global Model of the German Meteorological
Service and the measured wave spectra are taken from the NDBC (National Data Buoy Center, National Oceanic and Atmospheric Administration, USA).

The wave model was set up on the numerical mesh shown in Fig. 4 with a time step of 10 min. The resolution in direction space was set to 10° and the frequency bandwidth was chosen from 0.04Hz – 1.0Hz with totally 34 logarithmically distributed frequency increments. We have used the default source term formulation in the SWAN Model (C-I) and the alternative formulation like suggested by Makin & Stam with the above shown parameterization (C-III). Additionally we have accounted in separate calculations for the effects of attenuation (Att.) of fast running waves, the effect of opposing wind on the sea state (Opp.) and both effects (AttOpp.). In these calculations we set $A_W = 100$ and $c_{opp} = 25$.

The analysis of the results showed that as already expected that the default source term formulation in the SWAN model lead to strong underestimation of the wave period. Like already shown by Rogers et al. (2002) or Roland et al. (2004, 2005) for different regions. The reason for this underestimation is mainly due to the Whitecapping function of Komen et al. The statistical results for this event are summarized in Tab.1. It can be seen that the hindcast results are much better especially with respect to the average period.

Roland et al. (2004, 2005) compared the hindcasted and measured wave spectra at the Baltic Sea and it was shown that especially for the developing wind sea the low frequency wave energy is strongly underestimated and high frequency energy
Figure 4 Investigation Area for the real case computations. Position of the measurement buoys, depth distribution and computational mesh.

Table 1: Statistics of the integral wave parameters for C-I (left) and C-III (right) for the September period averaged in time and over all buoys shown in Fig. 4.

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<td>1.446</td>
<td>MEAN</td>
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overestimated by C-I. The alternative source term formulation C-III reduced this effect but mostly for the overestimation of high frequency energy in the low frequency part of the spectrum the wave energy is still underestimated, though not as much as with C-I (see Fig. 5).

For the case of the Sargasso Sea we have extracted the wave spectra for buoy 41010 (see Fig. 6) and have plotted below the differences between hindcasted and measured wave energy. It can be seen that especially when the hurricane wind fields are over the buoy (see shaded regions in Fig. 6), which means strong forced conditions, C-I tends to overestimate the high-frequency energy and gives similar signatures in the energy differences plots like for the case of the Baltic Sea (see Fig. 5 and 6), though in a different frequency region of the wave spectra as the waves at the Sargasso Sea are much longer. C-III leads in both regions with the same
parameterization to a similar effect on the hindcasted wave spectrum. The overestimation of high frequency energy is reduced, especially under strong forced wind conditions and the underestimation of low frequency is also much smaller.

For the case of the Sargasso Sea we have calculated spectral statistics (BIAS, RMS-Error and the Correlation coefficients) for all measurements during the investigated time period. The results are shown in fig. 7 and 8. At the left column of Fig. 7 the comparison with C-I and C-III are shown. It can be seen for the estimation of the spectral BIAS that C-I underestimates in average the measured wave energy. The correlations coefficients and RMS-Errors distribution in spectral space show clearly the advantage of C-III. However, it can be seen that still the low frequency energy is in the grand average underestimated by C-III. When thinking on design values for coastal engineering problems this results are still on the insecure side and need further revision.

For the calculations with the effects of wave attenuation in following and opposing wind conditions the calculations results are much less spectacular. The effects lead in generally to a stronger negative BIAS because for the same parameterization of the other source terms for this computation more energy is dissipated in total. For the case Att. we cannot see any improvement of the results only stronger negative BIAS, higher RMS-Errors and worse correlations in comparison to the reference run. For the Opp. case the negative BIAS is also greater then in the reference run but we have a clear signal of better correlations and smaller RMS-Errors, which are promising results.

![Figure 5 Differences between measured and hindcasted variance density as a function of time and frequency at the location of buoy 1 during the north-east storm Roland et al. (2005).]()}
Figure 6 Differences between measured and hindcasted wave spectra during the September period 2004. Gray shaded regions show the time when the Hurricane wind field is over or near by the buoy and the yellow shaded regions show a time period where the low frequency energy is under predicted by the wave models. Red line shows the measured peak period.

Especially when we see these results in the context that we haven’t tuned any of the involved parameters. It seems that the wave damping due to opposing wind can improve the hindcast results. For the AttOpp. case the results are worse for all statistical parameters. This is due to the strong additional damping which must be balanced in a new source term formulation. However, this new source term balance must be retuned and further verified in various field conditions in order to see whether there is really a clear benefit in accounting for these processes.
Figure 7: Spectral statistics for the comparison between C-I and C-III (left) and for the Att. case (right). The statistics are taken as an average over all measurements and buoys.
Conclusions

In this paper we could show that with C-III the hindcast results for the deep water improved significantly in comparison with C-I. The whitecapping function of Alves & Banner combined with the wind input function of Makin & Kudryavtsev (1999) is a better choice than C-I. In the latest version of the SWAN model the dissipation function of Alves & Banner can also be chosen as an alternative in a modified form (see Holthuijsen et al. (2005)). We suggest, especially for the estimation of design values for coastal engineering structures, to use rather the whitecapping function of Alves & Banner than the one of Komen et al. because it is crucial for the design of coastal structures to have a proper estimation of the wave kinematics. If already in the deep water region the average period is underestimated than in shallow water the shoaling will also be underestimated which leads to an insecure design of coastal structures. However the formulation of Alves & Banner is for sure not a final solution for this problem. Our results showed clearly that the low frequency energy is still consequently underestimated. Van der Westhuysen et al. (2006) showed results with the modified version of the Alves & Banner formulation like used in the newest SWAN version for the shallow lakes in the Netherlands with similar trends like presented in this study.

When we recall that the actual spectral balance is not a detailed one, in the sense that a lot of empirical values are entering the model formulation and the estimation of the nonlinear fluxes within the wave spectrum are also more or less crude approximation of the reality (though the best one that we have). We think that this parametric spectral balance should be retuned on the foundation of field experiments and in-situ measurements in various environments with the aim to reduce this underestimation of low frequency energy.

Especially the ongoing advances in the estimation and measurement of the dissipation rates in deep water are crucial developments in the near past (Babanin et al. (2005), Manasseh et al. (2005) and Young et al. (2005)) and the latest efforts by van Vledder (2006) concerning the effective estimation of the nonlinear fluxes are bringing new hope for a detailed spectral balance, tough it is still a long way to go.

Figure 8 Spectral statistics for the case Opp. (left) and AttOpp. (right).
References:


Donelan, M. A Nonlinear Dissipation Function Due to Wave Breaking, Proceedings


