Experimental Source Terms for Spectral Wave Forecast Models

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Abstract

A field experimental study of the spectral balance of the source terms for wind-generated waves in finite-depth water was carried out in Lake George, Australia. The measurements were made from a shore-connected platform at varying water depths from 1.2 m down to 20 cm. Wind conditions and the geometry of the lake were such that fetch-limited conditions with fetches ranging from approximately 10 km down to 1 km prevailed. The resulting waves were intermediate-depth wind waves with inverse wave ages, measured by the ratio of wind speed to the speed of the dominant (spectral peak) waves, in the range of \( 1 < \frac{U_{10}}{c_p} < 8 \). The atmospheric input, whitecap dissipation and bottom friction were measured directly and synchronously by an integrated measurement system. In addition, simultaneous data defining the directional wave spectrum, atmospheric boundary-layer profile and atmospheric turbulence were available. The observations were conducted over a three-year period from September, 1997 to August, 2000.

Results revealed new physical mechanisms in the processes of spectral dissipation of wave energy and small-scale air-sea interaction. New parameterisations of the wind energy input and wave energy dissipation are suggested. They are presented in forms suitable for spectral wave models. The new features of the source functions are particularly relevant when modelling the steep wave/strong wind conditions.

Energy balance of the source functions was verified by means of independent redundant checks. Results of implementation of the new source functions in a research spectral wave model will also be presented.

1 Introduction

Spectral evolution of the wind-generated wave field is commonly described by the radiative transfer equation (Hasselmann, 1960):
\[
\frac{dF(\omega, k)}{dt} = I(\omega, k) + N(\omega, k) + D(\omega, k) + B(\omega, k)
\]  

(1)

where the total derivative of the frequency (\(\omega\)) -wavenumber (\(k\)) spectrum \(F(\omega, k)\) on the left hand side is balanced by the sum of energy source \(I\), sinks \(D\), and spectral redistribution \(N\) terms on the right. In the finite depth, energy sink due to wave interaction with the bottom \(B\) is usually considered explicitly also. Here, only energy terms for wind input \(I\), dissipation in the water column \(D\), bottom friction \(B\), and four-wave non-linear interactions \(N\) are mentioned, as they are usually the dominant terms. Equation (1) is the basic equation used in most phase-average numerical wave prediction models.

A field experiment to study the spectral balance of the source terms for wind-generated waves in finite water depth was carried out in Lake George, Australia (Fig.1). The measurements were made from a shore-connected platform at varying water depths from 1.2 m down to 20 cm. Wind conditions and the geometry of the lake were such that fetch-limited conditions with fetches ranging from approximately 10 km down to 1 km prevailed. The resulting waves were intermediate-depth wind waves with inverse wave ages, measured by the ratio of wind speed at 10m height above the sea level, \(U_{10}\) to the speed of the dominant (spectral peak) waves, \(c_p\) in the range of \(1 < U_{10} / c_p < 8\).

The atmospheric input, whitecap dissipation and bottom friction were measured directly and synchronously by an integrated measurement system (Young et al., 2005). In addition, simultaneous data defining the directional wave spectrum, atmospheric boundary-layer profile and atmospheric and underwater turbulence were available. The contribution to the spectral evolution due to nonlinear interactions of various orders is investigated by a combination of bispectral analysis of the data and numerical modelling. The relatively small scale of the lake enabled experimental conditions such as the wind field and bathymetry to be well defined. The observations were conducted over a three-year period from September, 1997 to August, 2000, with a designated intensive measurement period (AUSWEX) carried out in August-September 1999. High data return was achieved (Young et al., 2005).
2 Wind Input

Nearly all of the momentum delivered from wind to waves comes about through wave-induced pressure acting on the slopes of waves - "form drag". Direct field measurements of the wave-induced pressure in air flow over water waves are difficult and consequently rare. In order to measure microscale oscillations of induced pressure above surface waves, a high-precision wave follower system was developed at the University of Miami, Florida and employed at Lake George (Donelan et al., 2005). The principal sensing hardware included Elliott pressure probes, hot-film anemometers and Pitot tubes. The wave-follower recordings were supplemented by a complete set of relevant measurements in the atmospheric boundary layer, on the surface and in the water body. The precision of the feedback wave-following mechanism did not impose any restrictions on the measurement accuracy in the range of wave heights and frequencies relevant to the problem. Thorough calibrations of the pressure transducers and moving Elliott probes were conducted. As a result of this study, it was shown for the first time that the response of the air column in the connecting tubes provides a frequency-dependent phase shift, which must be accounted for to recover the low-level induced pressure signal.

2-1 Wind Input Spectral Function

Previously reported measurements of the wave-induced air pressure are for deep water conditions and conditions in which the level of forcing is rather weak: $U_{10}/c_p < 3$. The data reported here, obtained during AUSWEX, have the range of $1 < U_{10}/c_p < 8$. The propagation speeds of the dominant waves were limited by depth and the waves were correspondingly steep. This wider range of forcing and concomitant wave steepness revealed some new aspects of the rate of wave amplification by wind - the so-called "wind input source function" in the energy balance equation for wind-driven water waves (1) (Donelan et al., 2006).

The dimensionless growth rate is customarily expressed in terms of the fractional energy increase $\gamma$, which is a spectral function

$$\gamma(\omega) = \frac{\rho_w}{\rho_a} \frac{1}{\omega F(\omega)} \frac{\partial F(\omega)}{\partial t}. \quad (2)$$

Here, $\rho_w$ and $\rho_a$ are densities of water and air respectively. Once the growth rate function $\gamma(\omega)$ is known and the power spectrum $F(\omega)$ is available, the dimensional wind energy input is

$$I(\omega) = \rho_a \omega g \gamma(\omega) F(\omega) \quad (3)$$

where $g$ is the gravitational constant.

Two new features of the wind-wave interaction were discovered as a result of the Lake George study. Firstly, it was found that the exponential growth rate parameter $\gamma$ depended on the slope of the waves, $ak$ ($a$ is the wave amplitude). Secondly, it was found that for very strong forcing a condition of “full separation” occurs, where
the air flow detaches from the crests and re-attaches on the windward face leaving a separation zone over the leeward face and the troughs. In a sense, the outer flow does not “see” the troughs and the resulting wave-induced pressure perturbation is much reduced, leading to a reduction in the wind-input source function compared to that obtained by extrapolation from more benign conditions. The two features affect the momentum transfer in opposing ways, increasing it in moderate forcing conditions and reducing it in strong forcing conditions. The validity of the parameterisation across the spectrum was verified by independent measurements of the integrated momentum flux across the interface.

The latter feature may have significant implications for descriptions of air-sea interactions at strong-wind conditions. This result, along with recently discovered by Donelan et al. (2004) effect of limiting value of aerodynamic roughness at extreme winds, point out to an important conclusion, not completely unexpected. The drag coefficient dependences obtained at moderate-wind conditions and then extrapolated into strong-wind situations may significantly overestimate the drag and air-sea momentum exchange.

The source function parameterised in terms of wave steepness and degree of separation is shown to be in agreement with previous field and laboratory data obtained in conditions of much weaker forcing and wave steepness. The strongly-forced steady-state conditions of AUSWEX have enabled us to define a generalised wind input source function that is designed to work in the entire range of wave generation by wind: from light and moderate to very strong winds; from young waves to mature seas. An analogue of this parameterisation, expressed in terms of the wave spectrum and suitable for use in wave spectral models, has form (Donelan et al., 2006):

\[ \gamma = G \sqrt{B_n} \left( \frac{U_{10}}{c} - 1 \right)^2, \]

\[ G = 2.80 - 1.00 \cdot \tanh(10 \sqrt{B_n} \left( \frac{U_{10}}{c} - 1 \right)^2 - 11). \]

Here, \( B_n(\omega) = \frac{\omega^5 F(\omega)}{2g^2} \) is the spectral saturation and \( A(\omega) \) is the directional spreading function defined in Babanin and Soloviev (1998).

### 2-2 Enhancement of the Wind Input due to Wave Breaking

In the finite-depth environment of Lake George, breaking waves play an important role in the momentum exchange between wind and waves. Direct evidences were found of the influence of wave breaking on the wave-induced pressure in the air flow over water waves, and hence on the energy flux to the waves (Young and Babanin, 2001).

In Fig.2, phase-resolvent plots of mean wave profile, mean wave-induced pressure with respect to the wave and mean energy-flux distribution over the wave are shown. The blue line signifies all waves, red line – non-breaking waves and green
line – breaking waves. The plots are a result of averaging over 6547 waves, 1133 of which were breaking. Evidently, the breaking waves are somewhat steeper on average (middle panel). The induced pressure over breaking waves (bottom panel), however, is significantly increased and its maximum is shifted towards windward wave face which fact further contributes to the enhanced wind-wave energy flux shown in the top panel.

These measurements allowed an assessment of the magnitude of the breaking-induced separation enhancement and provided a basis for parameterising the effect. Overall, this produced an enhanced wave-coherent energy flux from the wind to the waves with a mean value of 1.9 times the corresponding energy flux to the non-breaking waves. We propose that the breaking-induced enhancement of the wind input to the waves can be parameterised by the product of the non-breaking input and the breaking probability:

\[ \gamma = \gamma_0 (1 + b_r) \]  

(5)

where \( \gamma_0 \) is the spectral wave growth rate increment in absence of wave breaking and \( b_r \) is the associated breaking probability per wave period (ratio of number of breaking crests to the number of non-breaking crests).

Figure 2 Phase-average plots of (top) energy flux distribution; (middle) Mean wave profile; (bottom) Surface pressure distribution
2-3 Other Wind-Wave Interaction Effects

Further analysis of the wind-wave energy exchange revealed a fine-scale inhomogeneity of such input, both in time (over a few wave-period time scale) and in space (over a few wave lengths) (Agnon et al., 2005). Such oscillations appear to be correlated with fluctuations of wave skewness and asymmetry and have a potential to essentially alter the average input estimates if are properly accounted for. Oscillations of the wave field third moments (skewness and asymmetry) also relate strongly with the second most important mechanism of wave dynamics – wave dissipation due to breaking.

3 Wave Dissipation

Spectral wave energy dissipation represents the least understood part of the physics relevant to wave modeling (Babanin et al., 2006). There is a general consensus that the major part of this dissipation is supported by the wave breaking, but physics of this breaking process, particularly for the spectral waves, is poorly understood. Also, there is a growing evidence that the eddy-viscosity dissipation can be significant, particularly at the spectrum tail (e.g. Babanin and Young, 2005).

Theoretical and experimental knowledge of the spectral wave dissipation is so insufficient that, to fill the gap, spectral models have been used to guess the spectral dissipation function $D$ in (1) as a residual term of tuning the balance of better known source functions to fit known wave spectrum features. This is the only source function in (1) which has so far been inferred indirectly by modelling the evolution of the wave spectrum rather than by parameterising measured physical features directly.

Two different methodologies were used in the Lake George study to investigate the dissipation function. The first employed the acoustic noise spectrograms to identify segments of breaking and non-breaking dominant wave trains (Babanin et al., 2001). As a result, threshold-like behaviour of the breaking probability was revealed. If some characteristic wave steepness is below the threshold, the waves will not break (and whitecapping dissipation will be zero). If the steepness threshold is overcome, the breaking rates $T_b$ are proportional to the steepness excess over this threshold, all squared.

The average power and directional spectra for breaking and non-breaking waves were obtained by the segmenting method, and the difference was attributed to the dissipation due to wave breaking (Young and Babanin, 2005). This method provides an estimate of the spectral effects, both in frequency and directional domains, of the dominant wave breaking. The approach is illustrated in Fig.3 where difference between the spectra of breaking waves and broken waves is clearly seen and quantified.

As an independent second approach, a passive acoustic method of detecting individual bubble-formation events was developed. This method was found promising for obtaining both the rate of occurrence of breaking events at different wave scales and the severity of wave breaking (Manasseh et al., 2005). A combination of the two methods should lead to direct estimates of the spectral distribution of wave dissipation.
If the wave energy dissipation at each frequency were due to whitecapping only, it should be a function of the excess of the spectral density above a dimensionless threshold spectral level, below which no breaking occurs at this frequency as mentioned above. This was found to be the case around the wave spectral peak.

A more complex mechanism appears to be driving the dissipation at scales different to those of the dominant waves. Dissipation at a particular frequency above the peak demonstrates a cumulative effect, depending on the rates of spectral dissipation at lower frequencies. In terms of the dissipation function $D$ such an effect will mean a two-phase behaviour: $D$ being a simple function of the wave spectrum at the spectral peak and having an additional cumulative term at all frequencies above the peak (Babanin and Young, 2006). The following parameterisation of the dissipation term was suggested:

$$D(\omega) = -a_1 \rho_w g \omega((F(\omega) - F_{sw}(\omega))A(\omega))^n - a_2 \rho_w g \int_{f_{sw}}^{\omega} (F(q) - F_{sw}(q))A(q) dq$$

(6)
where $\omega_p$ is the spectral peak frequency, $a_i$ are experimental constants yet to be comprehensively obtained, $F_{\omega p}(\omega)$ is the dimensional threshold, and it is likely that $n=1$ (see Babanin and Young (2005) regarding details relating to the last three parameters mentioned).

The nature of the induced dissipation above the peak can be due to either enhanced induced wave breaking or additional turbulent eddy viscosity or both. Babanin and Young (2005) showed that there are indications that the turbulent viscosity becomes significant at small wave scales, where the cumulative term of the function (6) dominates. Once this is true, the dimensionless spectral threshold below which no dissipation occurs, may not be universal across the spectrum. This complex issue is now under further investigation.

Young and Babanin (2006) also compared directional spectra of the breaking and non-breaking waves whose difference should be indicative of the directional distribution of the dissipation. They showed that directional dissipation rates at oblique angles are higher than the dissipation in the main wave-propagation direction and therefore the breaking tends to make the wave directional spectra narrower. If confirmed, this conclusion may have very significant implications for the directional shape of $D$: unlike $I$, it would be bimodal with respect to the wind direction, and the main wave direction would be characterised by a local minimum of the directional spectrum of dissipation.

Another feature of the spectral dissipation function, revealed by the Lake George studies, shows peculiarity of the dissipation behaviour at strong winds (Manasseh et al., 2006). For the moderate wind, according to (6), the dissipation function is mainly determined by the wave spectrum. In these circumstances, the wind influence on wave breaking and energy attenuation is indirect: the wind changes the wave spectrum first, and this change brings about alterations of the breaking as a consequence. At strong winds of $U_{10} > 14m/s$, however, further increase of the wind speed and the wind input does not cause noticeable changes of the wave spectrum. The excessive wind input, or at least a significant part of that, appears to be dissipated locally through an enhanced breaking.

Figure 4 (left) Spectrum of breaking waves (blue) and broken waves (red); (right) Ratio of the two spectra
4 Wave-Bottom Interaction

Field observations carried out at Lake George were supplemented with laboratory experiments conducted in a wave flume and water tank at the Australian Defence Force Academy (ADFA), Canberra to accurately estimate the bottom friction term $B$ in eq.(1). Measurements in the bottom boundary layer were performed by means of acoustic and laser Doppler velocimeters, as well as by means of a specially-designed shear plate capable of measuring the bottom stresses directly. Given the substantial body of knowledge in this field for monochromatic waves, tests were conducted to create an empirical link between monochromatic and spectral conditions to enable this existing knowledge to be applied to spectral conditions. In the laboratory studies, for bed conditions ranging from smooth to rough and for JONSWAP-like wave spectra, it was found that, in terms of the loss of wave energy due to the bottom friction $B$, the spectrum can be represented by a monochromatic wave, the bottom-velocity amplitude of which is equal to $1.88u_{rms}$ and with a period equal to the spectral peak period (Mirfenderesk and Young, 2003).

Two additional sets of laboratory experiments were conducted to study conditions of ripple formation on a silt-covered bottom and to study the effects of direct injection of turbulence into the bottom boundary layer by breaking waves (Babanin et al., 2005). Bottom of the ADFA’s tank was covered by a layer of Lake George silt, and thus the bottom interaction term was estimated for Lake George silt-bottom conditions, and the estimates were independently verified by means of comparisons with other energy sources and sinks, simultaneously measured while in situ, which together had to satisfy the energy balance (1).

The friction factor for the Lake George silt was obtained in a series of experiments in the uni-directional flow tank. Variability of the bottom roughness and conditions of ripple formation in the fine-sediment silt were analysed. It was shown that, as the mean flow velocity grows, the friction factor plateaus until it abruptly increases 60 times once the ripples are formed.

Also, the effect of wave breaking on the bottom boundary layer was examined. When a wave breaks, particularly if it is a plunging breaker, it injects a turbulent jet into the water column. The jet has a vertical component, and if it can reach the bottom, it will enhance the turbulent mixing, thus reducing the vertical velocity gradient in the boundary layer and, correspondingly, the bottom stress $\tau_0$. The experiment was conducted at the ADFA’s wave flume where the waves were forced to break at a given location and $\tau_0$ was directly measured by means of the shear plate. The reduction was found to be of the order of 10%, and it may become noticeable in field conditions in case of frequent wave breaking at finite depths.

Overall, the bottom dissipation term $B$ was found to be an important part of the total energy balance in the finite depth wave field, measuring up to 20% of the total dissipation in extreme wave cases.
5 Total Dissipation in the Water Column and Consistency Check

As a consistency check for Lake George measurements and parameterisations, integrated wind input was compared with total dissipation measured independently. To obtain the total dissipation throughout the water column outside the bottom boundary layer, vertical profile of the volumetric rate of total kinetic energy dissipation $\varepsilon(z)$ was analysed (Babanin et al., 2005). This led to a new parameterisation of the depth distribution of $\varepsilon(z)$:

$$
\varepsilon(z) = \begin{cases} 
\text{const} & z \leq 0.4H_s \\
 z^{-1} & z > 0.4H_s, \quad U_{10} < 7.5 \\
 z^{-2} & z > 0.4H_s, \quad U_{10} \geq 7.5 
\end{cases}
$$

In this parameterisation, the total dissipation rate is a function of distance from the water surface $z$, significant wave height $H_s$, and wind speed $U_{10}$. The latter comes into importance due to the fact that once the waves start to break at $U_{10} > 7.5 \text{m/s}$, they significantly enhance the background turbulence level in the water column (Terray et al., 1996).

Comparison of the spectrum-integrated wind input $I$ with the total dissipation $T$ obtained as a sum of the bottom dissipation $B$ and depth-integrated dissipation $\varepsilon$ is shown in Fig.3. Given the uncertainties involved the agreement is quite good and supports the independent measurements of input and dissipation.

![Figure 5 Total dissipation in wave water column T versus measured total wind input I.](image_url)
6 Conclusions

Spectral terms for wind input $I$, whitecap dissipation $D$ and bottom friction $B$ (1) were experimentally approached in a detailed study at Lake George, Australia. The field study was supplemented by a series of laboratory experiments. A number of new physical features of behaviour of the source/sink functions were revealed. These features were parameterised in forms suitable for adopting in wave spectral models (i.e. eqs. 4-7 above). The parameterised spectral forms of the source terms/sinks have been tested in a research wave evolution model.

7 References


