Measuring Progressive Edge Wave by a Single Instrument: Applications in Tsunami and Typhoon Waves

Guan-Yu Chen    Hsiao-Ching Chien

ABSTRACT

If waves are generated in a restricted region far away along the coast, their direction is easy to determine. For a wave gage at a long and straight coastline, the measured wave can be classified into three categories, viz. direct plane wave from the source, short-crested wave reflected once by the shoreline, and progressive edge wave repeatedly reflected and refracted along the coast. Traditionally, an edge wave is measured along a long and straight coast with a couple of pressure-gage/current-meter arrays to acquire the frequency-wavenumber dispersion diagram. In the present study, flow-pressure relations for both infragravity edge and short-crested waves are used to identify the edge wave mode. This new approach is a useful tool if there is just one dominating edge wave mode and the edge wave is propagating away from a known source. Edge wave tsunamis and infragravity storm waves are analyzed as two examples.

1 INTRODUCTION

If waves are generated in a restricted region far away along the coast, their direction is easy to determine; although reflection of the shoreline is possible, these waves are basically progressive. Waves measured at a long and straight coastline can be classified into three categories, viz. direct plane wave from the source, short-crested wave reflected once by the shoreline, and progressive edge wave repeatedly reflected and refracted along the coast.

A submarine earthquake is an example for infragravity (IG, 0.004Hz - 0.04Hz) wave source with restricted area. IG waves can also be generated by typhoon waves in the coastal region and hence the location of typhoon indicates the direction of IG waves. IG Waves in these two incidences are usually larger than normal situations and hence are very important for coastal engineers.

Edge waves play an important role in the formation of crescentic bedforms (Bowen and Inman, 1971) and have been shown to be related to rip current formation (Bowen, 1969) and harbor resonance (Chen et al. 2004) in previous studies. Field measurements, however, are very rare.

In the infragravity frequency studies, edge waves comprise a major part of water surface fluctuation as do leaky waves (Munk et al. 1964). One important feature of edge waves is that they are trapped nearshore and hence their amplitude decays exponentially in the cross-shore direction. On the other hand, leaky waves are plane waves and the amplitude is quite uniform in the lateral direction.

The exponential-decay feature alone cannot be used to identify the existence of edge waves: a leaky wave also decays offshore due to shoaling. Besides, the cross-shore standing wave composed of both normally incident and reflected leaky waves is a Bessel function with oscillating amplitudes; the location of nodes and antinodes is close to that due to a higher mode edge wave (Guza, 1974). Therefore, measurement along the cross-shore direction is insufficient for detecting whether the infragravity
component is trapped or leaky.

As had been developed in Huntley et al. (1981) and Oltman-Shay and Guza (1987), the most common method of edge wave measurement now is based on the dispersion relation: A series of instruments are set up alongshore; the amplitude variation gives the longshore wavenumber spectrum which, with information of the temporal variation, gives the spectrum over points in the frequency-wavenumber diagram. These points indicate the mode of an edge wave if they are located on the frequency-wavenumber curve plotted according to the dispersion relation of a specific edge wave mode.

For standing edge waves, the amplitude depends on the measurement location. Alongshore deployed instruments give observations near the nodes and the antinodes; hence, both propagating and standing waves can be treated. The longshore wavenumber spectrum gives the energy distribution among each wavenumber. Therefore, the dispersion diagram approach is applicable even if all possible edge wave modes exist at the same time. Therefore, the dispersion-diagram approach is applicable for either multiple edge wave modes or standing edge waves.

2. RESEARCH METHOD

Conventionally, separate instruments are used to measure water surface and fluid velocity. Both the acceleration of a buoy in the water or the pressure at the bottom can be used to obtain the water surface; the latter is more applicable for infragravity motion because the natural oscillation of the buoy can contaminate the recorded acceleration.

An acoustic wave approach has been proven accurate and convenient in measuring surface elevation employing the elapsed time of propagation, and the fluid flow from the Doppler shift effects. In an acoustic instrument, these functions can be lumped together; hence, both surface elevation/bottom pressure and flow information can be measured by a single instrument. The approach proposed in the present study is to utilize the special relation between flow and pressure, which is especially applicable to acoustic devices because just a single instrument alone can provide sufficient information for infragravity wave analysis.

Based on a scale-analysis, the nonlinear advection, Coriolis and viscous effects of the momentum equation are all negligible for infragravity (0.004Hz - 0.04Hz) motion. The momentum equation for a water particle can be represented by a simple balance between local acceleration and the pressure gradient:

\[
\frac{\partial}{\partial t} \ddot{u} = -\frac{1}{\rho} \nabla p. \tag{1}
\]

As a first approximation, specific Fourier components of dynamic pressure and x-velocity are both proportional to exp(ikx-i\omega t) for waves propagating along the x-axis; hence, equation (1) can be written as

\[
P = \rho \frac{\omega}{k} U \tag{2}
\]

where the capital quantities U and P are the amplitude of x-velocity and pressure variation, \(\rho\) the water density, \(\omega\) the angular frequency and \(k\) the wavenumber. Therefore, for an infragravity wave, the pressure amplitude is proportional to the velocity in its
propagation direction. The proportionality constant includes the phase velocity \( \frac{\omega}{k} \) which depends on the dispersion relation of the specific water wave.

For an infragravity leaky wave, the wavelength is much longer than the water depth. Its phase velocity is exactly the shallow water celerity \( \sqrt{gh} \), where \( h \) is the local water depth, and the maximum velocity is in the wave direction \( x \). Hence, the pressure amplitude is related to the maximum velocity through

\[
P = \rho \frac{\omega}{k} U = \rho \sqrt{gh} U
\]

(3)

For an edge wave propagating along the shoreline parallel to the \( x \)-axis, the dispersion relation is (see Eckart, 1951 or Mei, 1989)

\[
\omega^2 = gk\alpha(2n+1)
\]

where \( \alpha \) is the constant bottom slope (see Fig. 1(b)) and the integer \( n \) is the mode number. Substituting the dispersion relation, the momentum equation in the longshore \( x \)-direction, equation (2), can be further simplified and a relation between the pressure amplitude and the amplitude of longshore velocity in its propagation direction is obtained:

\[
P = \rho \frac{\omega}{k} U = \rho \frac{g\alpha(2n+1)}{\omega} U
\]

(4)

That is, for an edge wave of mode \( n \) propagating alongshore, the pressure is proportional to the longshore velocity \( U \). Note that for simplicity the bottom slope used here is the tangent of the sloping angle instead of the sine function used in Ursell (1952). In practice, the difference between sine and tangent functions is indistinguishable even if the bottom slope is as large as 15 degrees.

It should be noted that the dispersion relation of edge wave used in equation (4) is based on the shallow water equation which is a very good approximation for infragravity waves in the nearshore area where the measuring instrument is usually deployed. In case a measurement is done in the deep ocean, the modified dispersion relations can be found in Ursell (1952).

The flow-pressure relation for a short-crested wave can be derived similarly (Chen, 2008). With the same longshore wavenumber and angular frequency, its flow-pressure relation is exactly the same as that for an edge wave. Therefore, we do not have to distinguish these two kinds of waves because their behavior is similar.

### 3. THE FLOW-PRESSURE RELATION IN TYPHOON WAVE OBSERVATIONS

#### 3.1 Non-stationary data and Hilbert-Huang transformation

Tsunami cannot occur before the rupture of the tsunamigenic earthquake and hence is non-stationary. To analyze non-stationary information, an adaptive approach called Empirical Mode Decomposition (EMD) was developed by Huang et al. (1998). In this decomposition process, a riding wave called the Intrinsic Mode Function (IMF) is obtained through repeated siftings; then, the Hilbert spectrum \( Y(t) \) of each IMF \( X(t) \) is obtained by the Hilbert Transform

\[
Y(t) = \frac{1}{\pi} \text{p.v.} \int_{-\infty}^{\infty} \frac{X(t')}{t-t'} \, dt'
\]
where p.v. means Cauchy principal value. $X(t)$ and $Y(t)$ form a pair of complex conjugates in $Z(t)$

$$Z(t) = X(t) + iY(t) = b(t)e^{i\theta(t)} \quad (5)$$

where $b(t)$ is the amplitude used in the spectrum analysis and $\theta(t)$ is the instantaneous phase angle. The Hilbert Huang Transform (HHT, Huang et al., 1998) is the process of calculating the Hilbert transform of each IMF which is obtained via the EMD process and is eliminated from the original time series before another EMD process begins.

Besides HHT, the wavelet analysis is also applied to analyze non-stationary information and is very popular. However, HHT is used in the present study because it is an adaptive approach and is easy to apply.

3.2 Analysis of Typhoon Waves outside Hua-Lien Harbor

Recently, the Center of Harbor and Marine Technology (CHMT) measured both pressure and flow for months near the entrance of Hua-Lien Harbor on the east coast of Taiwan; the locations of the three stations are shown in Figure 4. The operation of Hua-Lien Harbor has long been disturbed by typhoon waves even when the typhoon is one thousand kilometers away. This amplified oscillation inside the harbor has been explained by resonance induced by infragravity edge waves (Chen et al. 2004) and hence the CHMT measurements are important in clarifying the importance of edge waves.

The observed flow-pressure relation in the infragravity range shown in Figure 2 demonstrates the existence of edge waves. The information is measured during the period of Typhoon Kaemi (2006/7/23 – 2006/7/26) shows significant feature of Stokes edge wave. The figures shown are taken in the first eight successive hours. Diagrams in the other time of this typhoon show similar edge wave features. These diagrams provide a solid evidence for the existence of edge waves outside Hua-Lien Harbor.

4. EVIDENCES OF AN EDGE WAVE IN THE HENG-CHUN TSUNAMI

Tsunamis have very high impact, but tsunami incidents are very rare. To prepare for a future tsunami, studies of small incidents, such as the tsunami induced by the 2006 Heng-Chun earthquake, are very important because these studies can increase our understanding of tsunami features.

One of the most important features in tsunami propagation is the generation of edge waves. A tsunami from the deep ocean incident obliquely to the coast, or a tsunami generated near the coastline can be trapped by the coast and form edge waves (e.g. Fuller and Mysak, 1977; Gonzalez et al., 1995; Fujima et al., 2000; Lynett and Liu, 2005 and references cited therein). The arrival time of a tsunami will lag dramatically if edge waves are generated because the bathymetric refraction of edge waves in the shallow nearshore region takes much longer time than the direct propagation from the wave source through the deep ocean to the coast at risk.

For example, the arrival time of the Kamchatka tsunami in 1952 were found to be twice longer for some places. Similar phenomena had been observed in the Iturup tsunami in 1963 (Hatori and Takshashi, 1964). More recently, the 25 April 1992 Cape
Mendocino earthquake generated a tsunami which was recognized as a Stokes edge wave in Crescent City (Gonzalez et al., 1995) and the largest tsunami wave arrived almost 3 hours after the first wave. This indicates that an edge wave might arrive unexpectedly several hours after the first tsunami waves have subsided; therefore, when edge waves are generated in tsunami incidents, the total duration will become much longer and the region at risk should be prepared for both direct tsunami and edge wave tsunami components. Besides the propagation time, edge waves can also affect the tsunami height distribution. An edge wave has its maximum amplitude at the shoreline; it is possible that this amplitude is larger than the direct wave. In the 1992 Cape Mendocino tsunami, wave amplitudes 174 km away at Crescent City were about twice of that observed at North Spit which is located between Crescent City and the wave source at Cape Mendocino and the distance to the source is only 59 km. Numerical experiments of Lynett and Liu (2005) also show that in some cases the secondary run-up peaks due to the propagation of edge waves is larger than the first peak.

The surface elevation of the tsunami simulated by COMCOT numerical model is first compared with measured elevations at various tidal stations (Chen et al. 2008) at Kaohsiung, Dong-Gang, Siao-Liou-Ciou, Syun-Guagg-Zuei along the west coast of Taiwan; with their locations indicated in Fig. 1. The comparison is reasonably good, as is shown in figure 5 which is the comparison at Kaohsiung. The snapshots in Fig. 6 clearly exhibit the feature that oscillation is trapped nearshore. Based on the numerical simulation, an edge wave seems to play a very important role in the Heng-Chun tsunami. Further investigation of the pressure-flow relation will be presented in the workshop.

ACKNOWLEDGMENTS

This research was completed with grants from the Water Resource Agency of Taiwan, Republic of China, under contract MOEAWRA0970305. We thank Mr. H.M. Tseng, CHMT, for providing the field measurement.

REFERENCES

Generated by the Mw 7.8 1906 San Francisco Earthquake. Geology 27(1), pp.15-18.


Fig1. Tracks of typhoon in the east of Taiwan during 2005-2006 taken from the CWB website.
Fig 2. Pressure-flow relation in the period of Typhoon Kaemi (2006/7/23—2006/7/26) shows feature of Stokes edge wave. These figures are taken in the first eight successive hours.

Fig 3. Bathymetry of southwestern Taiwan
Fig4. Numerical simulation of the surface elevation at Kaohsiung. The red boxes are the internals when the field measurement is available.

Fig5. Comparison of numerical simulation and the field measurement of Kaohsiung of the first two hours.

Fig6. Propagation of edge wave tsunami in numerical simulation