Simulation of Nonlinear Wave Propagating over Periodic Seabed by Using Multi-layer Boussinesq Model

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ABSTRACT

Numerical experiments based on multi-layer Boussinesq model developed by Lynett & Liu (2004) were performed to study nonlinear wave propagating over periodic seabed. Both one-layer and two-layer models were used. Our results indicate that no visible difference between one-layer and two-layer model is found. Furthermore, to understand the nonlinear effect, the reflection coefficients due to periodic seabed are discussed for two different incident wave heights. Three methods including Goda & Suzuki (1976), Mansard & Funke (1980) and Lin & Huang (2004) for separating incident and reflective waves are adopted and compared. The results shows that it is necessary to include higher order term (nonlinear term) under certain circumstances where the nonlinear interaction is important. When the vertical wall (total reflection) is considered at transmission side, it is found that the displacement at the wall can be as large as 5 times of incident wave height even under Bragg resonance conditions. This result suggests that the idea by using artificial periodic bars to protect the shoreline may need further consideration.

1. INTRODUCTION

The focus in protecting of the traditional seacoast is on the hard structure which is built directly nearby the seashore such as the sea wall, the jetty, the mole and the artificial reef. However, few studies have been done on the seacoast landscape and the water affinity demand in the shore. Recently, the Boussinesq equation has become the most popular equation that governs wave transformations. Boussinesq (1872) derived the original Boussinesq equations. Thereafter, numerous researchers improved and extended their applicability. Madsen et al. (1991) improved the Boussinesq equation to enable it to be applied to relatively deeper water. The improved Boussinesq equation has a better dispersive characteristic than the conventional Boussinesq equation. Nwogu (1993) derived a Boussinesq equation with velocity at arbitrary depth as a dependent variable. According to the study, He derived the same dispersion relation as did in Madsen et al. (1991). Wei and Kirby (1995) numerically solved the equation derived by Nwogu (1993) using a fourth-order Adams-Bashforth-Moulton predictor-corrector scheme efficiently reducing the errors associated with the numerical calculations. The model is the well-know WKGS model. Lin et al. (2004) derived a set of highly nonlinear Boussinesq-type equations, which simulate the propagations of waves that are affected...
by currents. All other nonlinear terms in the equations are maintained to describe more accurately of currents on wave transformations.

2. THEORETICAL BACKGROUND

2.1 The Multi-layer Boussinesq Model

The chapter introduces the governing equation, derived by Lynett & Liu (2004) which is in depth of water surface displacement of transmission of \( \zeta(x', y', t') \), and \( h(x', y', t') \), assume n layers of vertical horizontal height is \( d_n \) (as vertical length yardstick), \( h_n \) is water of characteristic depth, characteristic length of the wave \( l_0 = 1/k \) is the yardstick of horizontal length, \( t_0 / \sqrt{g h_0} \) is the yardstick of time, the characteristic of wave amplitude \( a_0 \) moves as the wave, we can define the dimensionless numbers:

\[
(\zeta(x', y', t')) / a_0 = \zeta' / a_0
\]

\[
h = h' / h_0 \quad , \quad \zeta = \zeta' / a_0
\]

\[
\eta = \eta' / h \quad , \quad (U_x, V_y) = (U_x', V_y') / (e_0 \sqrt{gh_0})
\]

\[
W_x = W_x' / \{e_0 \mu \sqrt{gh_0}\}
\]

(2.1-1)

The parameters of non-linear \((\varepsilon_0)\) and dispersion \((\mu_0)\) is:

\[
e_0 = a_0 / h_0 \quad , \quad \mu_0 = h_0 / l_0
\]

Assume the fluid did not have viscous effect, the continuous equation and momentum equation can be expressed as follows:

Continuous equation:

\[
\frac{1}{e_0} \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \left( \frac{h}{h_0} \eta_x \right) + \frac{\partial}{\partial y} \left( \frac{h}{h_0} \eta_y \right) = \frac{\partial}{\partial x} \left( \frac{h}{h_0} \eta_x \right) + \frac{\partial}{\partial y} \left( \frac{h}{h_0} \eta_y \right) + \frac{1}{6} \frac{d}{dx} \sum_{n=1}^{N} \frac{h_n}{h_0} \eta_n \left( \frac{d \eta_n}{dx} \right)^2
\]

\[
\frac{1}{e_0} \frac{\partial \zeta}{\partial t} + \frac{\partial}{\partial x} \left( \frac{h}{h_0} \eta_x \right) + \frac{\partial}{\partial y} \left( \frac{h}{h_0} \eta_y \right) = \frac{\partial}{\partial x} \left( \frac{h}{h_0} \eta_x \right) + \frac{\partial}{\partial y} \left( \frac{h}{h_0} \eta_y \right) + \frac{1}{2} \frac{d}{dx} \sum_{n=1}^{N} \frac{h_n}{h_0} \eta_n \left( \frac{d \eta_n}{dx} \right)^2 + \frac{2}{2} \frac{d}{dx} \sum_{n=1}^{N} \frac{h_n}{h_0} \eta_n \left( \frac{d \eta_n}{dx} \right)^2
\]

Momentum equation

(2-1.2)
2.2 Wave Reflection

The paper uses Goda & Suzuki (1976), Mansard & Funke (1980), Lin & Huang (2004) to calculate the reflecting, in accordance with computing technology of regular wave and irregular wave as follows.

Lin & Huang (2004) defines the high-order composition wave are different with Goda & Funke. Goda & Funke suppose all components are free waves. Lin & Huang build on the basis of regular wave separate the limitation waves and freedom waves into all components and calculate the reflection rate respectively. The theory can consult Lin & Huang (2004). Goda & Suzuki (1976) useing two gauges to separate the incident wave and reflection wave. Mansard & Funke (1980) applicates the method of least square and three-gauge method to separating the reflection wave according to the theory of Goda. Lin & Huang (2004) utilize the theory of Funke, write as the matrix of a 4x4 with method of least square.

2.3 Verification of the Numerical Model

In order to verify the accuracy of the measurement, the experiment is carried out in THL (Taina Hydraulics Laboratory). The experimental flume has a useful length 18m, and an effective width of 0.5m. The wave amplitude is 0.025m, water depth is 0.15 m, wave periods is 1.05s. In experimen, the frequency is 50Hz, and numerical frequency is 104.93Hz. Comparativing experimental and numerical value, the result is very similar(Fig.1). Besides, the reflection for waves over the regular sinusoidal seabed, which are compared to Chou & Hsu (2003). The wave condition: The wave amplitude is 0.01m, water depth is 0.156m, wave periods range from 0.7805s to 3.2655s. Except that in reflecting rate nearby 2S/L=1 is slightly small, the others cases have better to closely meet (Fig.2).
3. METHODS OF THE NUMERICAL MODEL

In order to investigate the phenomenon resonably and coordinate the experimental location arrangement, the article is divided into two fore mentioned situations. One of the main part is to investigate the multi-layered comparison in the model and compare with another experimental value. The next part is to compare the changeable situation between the reflection before the bars and the wave energy after the bars, as shown is the following statement.

As first, the waves are according to group C and D as Table 1. Two patterns, one-layer wave and two-layer simulate on multi-layer Boussinesq model through the regular seabed. When the waves produce Bragg resonance, draw the water level sequence chart. Further, compare the differences between two patterns. Afterward compare the similar wave value to the experimental value.

The reflection ratio before the bars, its arrangement is shown in Figure 3, and its wave condition is as shown in Table 1. It is divided into group A and B. each group according to different cycle divide into 53 units to discuss. The effect which resonates according to Bragg resonance describes: when incidence wave length (L) is submerged breakwater spacing (S) two times will produce resonance region. Therefore, 2S/L between 0.5~2.5, after the value result, it is found that the waveform tend to be stable about 50s later. Therefore, this article takes the time section from 60s to 80s to calculate the reflectance.

The method to carry out the study was using three kinds of analysis, which included Goda & Suzuki, Mansard & Funke, Lin & Huang methoddogy. To discuss the
differences which were produced by the high-frequency waves. The spectrum analysis is to pick out the certain time section ($\Delta t$) and transfer to discrete data. Then, transforms with the FFT obtained frequency spectrum.

Finally, turn the original sponge layer into a vertical wall after the same seabed to discussion, the wave energy change. The gauging wave on vertical wall water level change (non-breaking wave condition), and wave condition like table 1(C, D) show. Making the X2 outset length is 0.2m, afterward moves the vertical wall, each time moves 0.1m, to the X2=15m, altogether 316 numbers of classes according. For the reason that after the bars is erected on the vertical wall. Therefore, the time of each group which the wave shape stabilization needs not identical, but needs above at least 300s. Cuts this profile of water level use zero-up method analyzes the height of component waves. Furthermore, to discuss the relations between changes of water level on vertical wall and motion subemerged. Because using the condition will produce explicit Bragg resonance in the bars place to resonate ($2S/L=1$), therefore, may take advantage of the discussion when resonance is happened. To discuss situation of the wave energy behind the bars.

Table 1 The wave condition for numerical simulation wave passing through the regular seabeds and the different distance behind the bars

<table>
<thead>
<tr>
<th>Case</th>
<th>High (m)</th>
<th>Depth (m)</th>
<th>Period (s)</th>
<th>$Ur(HL^2/d^3)$</th>
<th>H/L</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>0.01</td>
<td>0.156</td>
<td>3.266~0.781</td>
<td>42.15~1.69</td>
<td>0.0025~0.0125</td>
</tr>
<tr>
<td>B</td>
<td>0.025</td>
<td>0.156</td>
<td>3.266~0.781</td>
<td>105.36~4.21</td>
<td>0.00625~0.03125</td>
</tr>
<tr>
<td>C</td>
<td>0.01</td>
<td>0.156</td>
<td>1.679</td>
<td>10.536</td>
<td>0.005</td>
</tr>
<tr>
<td>D</td>
<td>0.025</td>
<td>0.156</td>
<td>1.679</td>
<td>26.341</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

Fig 3. Numerical model arrangement, submerged high 0.05m, bars of interval spacing 0.1m, erects 6 gauges location are 3m, 8.68m, 8.88m, 9.08m, 9.28m, 24m.

4. RESULTS AND DISCUSSION

4.1 The Results of the Multi-layer Boussinesq Model

In order to understand the water level relation between one, two layers of the multi-layer Boussinesq model, the heights chosen 1cm and 2.5cm, the period is
0.156s and water depth is 0.156m accords with the greatest resonance condition (2S/L= 1).

We can find to pursue the wave condition is under a linear wave condition(see Fig.4-a, 4-b ).To use one layer is as nearly self-same on the wave form as two layers in the multi-layer Boussinesq.And with non-linear increase, the water level does not have big difference each other. In Fig.5,for being compared with the experiment, there is no obvious difference with layers between provable experiment value and model.So the study adopts the one layer of ways in the multi-layer Boussinesq model.

4.2 The Discussion of the Reflection

To understand the nonlinear effect, the reflection coefficients due to periodic seabed are discussed for different incident wave heights. Three methods including Goda & Suzuki (1976), Mansard & Funke (1980) and Lin & Huang (2004) for separating incident and reflective waves are adopt and compared.

In fig.6(a,b),the initial wave height is 0.01m, depth of water is 0.156m, the period is 3.266~0.7805s, the interval of the bars is S=1m, the height of the bar is 0.05m. Obviously, there is difference value when the reflecting rate is fetched to hight order.When the 2S/L ≈ 0.6 and 2S/L ≈ 2 ,the major affect is the the method of Lin & Huang(2004) for separating between bound wave and free wave.Fig.7-a and Fig.7-b show the spectral of the wave for the condition of the 2S/L=0.62 and 2S/L=2.

In order to seek the non-linear factor, the initial condition change with the 0.025m of wave height, 0.156m of depth of water, the 3.266 s ~0.7805s of the period, the interval of the bars is S=1m, the height of the bar is 0.05m. Obviously to strengthen non-linearly, Fig.8 show the Us=68.525 and Us=6.575. For 2-order and 3-order values, we find the incidence amplitude of using Goda & Suzuki are 8.8×10^{-3}m, 3×10^{-3}m and the reflection amplitude are 1.3×10^{-3}m, 5×10^{-4} m. The incidence amplitude of using Mansard & Funke are 8.9×10^{-3}m, 2.9×10^{-3}m and the reflection amplitude are 9×10^{-4}m, 6×10^{-4} m. When using the method of Lin & Huang, the component waves of high order separate two kinds, that is to say bound wave and free wave. According the reason, the incidence amplitude of 2-order are 2.92×10^{-2}m, 1.97×10^{-2}m and the reflection amplitude are 1.18×10^{-2}m, 1×10^{-2} m. The incidence amplitude of 3-order are 4.9×10^{-3}m, 2.5×10^{-3}m and the reflection amplitude are 1×10^{-4}m, 2×10^{-4} m.

4.3 The Wave Energy Behind the Bars

Fig.9-a show the conditions of the initial wave with the 0.01m of wave height, 0.156m of depth of water, the 1.679s of the period (Ur=10.536). The x-coordinate is the ratio behind the bars to vertical wall and the incident wave length (L=2m). As the result, when moving the distance with vertical wall to the submerged, the value of wave height of the maximum from water gauges of the vertical wall show regular variation. Furthermore, the interval distance between peak is L/2. In this study, the maximum values fall in about 4m-7m behind bars. Fig.9-b show the conditions of the initial wave with the 0.025m of wave height, 0.156m of depth of water, the 1.679s of the period(Ur=26.341). When the nonlinear effects is increase, the wave height become irregular.
Over and above, we can find wave energy to gather behind the bars. In conclusion, if we want to protect the shoreline by using artificial periodic bars, it should be considered at transmission side.

Fig 4. The time series of one layer and two layers
(a: case C; b: case D)

Fig 5. The results of the compare with experiment and the multi-layer Boussinesq model
Fig 6. (a, b) The reflection coefficients of 2-order and 3-order in regular initial waves. (case A, case B)

Fig 7. (a, b) The water level and wave spectrum in stability. (a: 2S/L=0.62; b: 2S/L=2, H=1cm, case A)
Fig 8. (a, b) The water level and wave spectrum in stability. (a: 2S/L=0.62; b: 2S/L=2, H=2.5cm, case B)

Fig 9. The dimensionless with wave height and distance behind the bars. (a: case C; b: case D)

5. CONCLUSIONS

We can conclude with certainty that the discussion by reviewing the reflection coefficients due to periodic seabed are discussed for two different incident wave heights and the vertical wall (total reflection) is considered at transmission side by using multi-layer Boussinesq model. Four of these findings are worth summarizing.

Taking the wave height as 1cm, 2.5cm, periods 0.156s, water depth 15.6cm to make the waves, and it corresponds to the maximum of resonance condition (2S/L=1). As the results, we can find that the differences between one-layer and two-layer using in model are not obvious.
For the reflection problems before the bars, making small amplitude, Lin & Huang’s 2-order and 3-order synthesized reflection ration tendency are the same. Lin & Huang can separate the bound wave and the free wave, its reflected value and Goda & Suzuki, Mansard & Funke in 2S/L 2 have the obvious difference. Moreover, making the wave in the big scale amplitude, the reflection in 3-order is more obvious than in the 2-order, because of more non-linear influences.

Between the bars and the vertical wall relations, it is found that when making small amplitude, change with the distance behind the bars. Measuring the wave height on the vertical wall. The interval of bars peck is L/2. We find the wave energy presents a periodic variation. Moreover, when making big amplitude, the interval of bars peck is the same, presents a irregular variation. The ratio between the initial wave height and distance behind the bars is from 4 to 7.5. When closing to the top wave position, the wave affected by the non-linear influences and reduce its energy.

To sum up, whether the initial wave is linear wave or not, height measured value on vertical walls is perhaps serveral times height to the initial wave. Consequently, by using artifical periodic bars to protect the shoreline may need further consideration.

REFERENCES

